Demand and Accumulation in Long-Run Capitalist Development: Unbalanced Growth and Crisis

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Abstract

This chapter examines how growth unfolds through unbalanced accumulation and how that very imbalance gives rise to stagnation and crisis tendencies. It develops a framework in which the expansion of productive capacity and capital accumulation at different speeds, shaped by technological change and distributive conflict. While accumulation generates the demand that reproduce it through time, it does through systemic dis-proportionalities: the transformation of capacity often outpaces, or lags behind, the conditions that make its realization possible. The analysis formalizes this tension as the central mechanism driving long-run dynamics, identifying domains where accumulation stabilizes and others where it turns into stagnation, partical crisis and regime crisis. By tracing how imbalance reproduces itself through the same processes that propel growth, this article re-frames crisis not as disruption but as the normal expression of accumulation. The framework links long-run demand and accumulation to structural crisis and set the stage for comparative analysis of accumulation regimes in future research.

1 Introduction

From the early XXI century the comparative political ecoomy field was dominated by the "Varieties of Capitalism" (VoC) approach, which place its theoretical underpinnings to explain cross-national diversity in terms of institutional complementarities between firms, labour, and the state. In the canonical formulation by Hall and Soskice (2001), the central contrast between liberal and coordinated market economies is featured by supply-side institutional arrangements such as corporate governance, wage bargaining, skill formation, argued as the theoretical underpinnings that explain distinct sets of competitive advantage. This research agenda was motivated to explain the different features of advanced capitalist economies and their contrasting institutional landscapes: mainly the United States and Western European countries. Subsequent extensions of this framework introduced additional types such as hierarchical and networked market economies identifying Latin American oligarchic legacies and East Asian dense business network-based industrialization (Schneider, 2008, 2009). Despite these typological extensions, the analytical focus remained on how institutional configurations shape the supply side of the economy and its capacity to deliver competitiveness without a particular concern on the insertion of national economies into the world-market, a feature that becomes particularly weak theoretically and empirically once looking at peripheral capitalist economies where economic growth is highly driven by export performance.

During the last decade though, CPE scholars driven to explain "varieties within varieties" have turned the agenda towards a demand-led approach, as argued extensively by Baccaro and Pontusson (2016); Baccaro et al. (2022); Baccaro and Pontusson (2023, 2025) through the conceptualization of "growth-models", have pushed CPE into a direction more concerned with macroeconomic dynamics, path-dependent trajectories, and distributive conflict to explain institutional and political developments. Drawing explicitly on long-standing post-Keynesian and heterodox macroeconomic traditions, the growth-model literature re-center the analysis of CPE under Keynes (1936) princple of effective demand institutional conditions under which wage-led and profit-led configurations emerge. Neo-Kaleckian models of growth and distribution provided the basic macroeconomic backbone, offering a tractable way to connect economic growth and income distribution to institutional features of wage bargaining, financialization, and welfare states. The field has thus moved from a purely supply-side institutionalism towards a demand-led framework that studies distinctive demand regimes.

Yet two weak flanks remain in this otherwise major advance. The first is spatial and historical. The growth-models literature has so far engaged only superficially with debates on geographically uneven development and the variegated character of capitalism as national economies are differentially integrated to the world-market in a hierachically organized world of states (Peck and Theodore, 2007; Jessop, 2015). By treating national growth models as largely self-contained regimes, it risks underplaying how accumulation in the core and the periphery are interrelated through trade, finance, and imperial infrastructures. What is at stake at the frontier of the debate in the literature of CPE is not merely identifying "varieties within varieties" of capitalism, but theorizing a variegated capitalist world market in which national demand regimes are constantly restructured by global hierarchies of money, production, state and corporate power which the field is advancing as a consensus aiming to "bridge the gap" between International and Comparative Political Economy (Nölke, 2023).

The second weak flank is theoretical and concerns the core assumptions of Neo-Kaleckian growth models, which underpin the demand-led turn of CPE of the last decade. While central to post-Keynesian macroeconomics, these models face fundamental issues in reconciling long-run convergence of capacity utilization with empirically grounded investment behavior. Skott (2012) remarks that the standard Kaleckian investment function either generates instability or requires implausibly adaptive changes in firms' desired utilization rates. Comlpementing this critique empirically, Skott and Zipperer (2012) find no empirical support for a Neo-Kaleckian specification for an investment function for the US case, rather, the evidence suggests to support alternative specifications in which investment adjust rapidly to changes in profitability and output adjusts slowly, in detriment of the Kaleckian framework where the utilization rate adjusts while investment remains relatively rigid. Researchers working under Neo-Kaleckian framework have tackled this critiques seriously as for example the works by Nikiforos (2013, 2016b, 2021, 2023) aiming to resolve these theoretical and empirical short-comings with micro-foundations and new data on capital utilization. While critical inquires to Nikiforos' works for consistency of the Neo-Kaleckian toolkit to study growth and distribution, other authors have embraced critiques by including autonomous expenditures (Lavoie, 2016), as suggested by proponents of the Sraffian Supermultiplier framework (SSM) (Pariboni, 2016; Gahn, 2023)

The SSM has emerged as an alternative demand-led framework that addresses part of these concerns by shifting the focus to long-period effective demand and the role of autonomous components in pinning down the trend growth rate (Serrano, 1995; Serrano and Freitas, 2017). In the SSM, normal capacity utilization is treated as a exogenous, being defined by the institutional and technical conditions of production based on the works of (Sraffa, 1960) and later interpretations reconciling classical political economy with Keynesian principle of effective demand Garegnani (1978, 1979), by extending effective demand to the long-run identifying its long-run trajectory as driven by "autonomous" components of aggregate demand, i.e. macroeconomic expenditures defined by not being induced by distributive conflict neither being directly generative of productive capacities such as gross capital formation does. This structure has obvious appeal for Comparative Political Economy: it invites us to read growth regimes as historically specific configurations of autonomous demand—welfare states, export complexes, national innovation investment systems—stabilized or destabilized by institutions that mediate class conflict and external constraints. Recent contributions have begun to deploy the SSM explicitly within the growth-models debate, using it to reinterpret varieties of capitalism as varieties of demand-regime rooted in distinct autonomous demand trajectories (Passos and Morlin, 2022; Morlin et al., 2024).

However, the SSM-based turn in Comparative Political Economy also inherits its own set of tensions still unresolved within post-keyensian debates. At the macroeconomic level, the notion of autonomous demand has been criticized for assuming too much independence of these components from income distribution, financial conditions, and external imbalances. For example, Skott (2017) cast doubt on whether consumption credit demand or other allegedly autonomous demand components can be treated as exogenous in the long run without smuggling in implausible assumptions about the order of magnitude as share of output. In my own interpretation, politically, the SSM framework leans the embedded-autonomy understanding of the state in capitalist development (Evans, 1995), rather than as a social relation as suggested by contemporary state theory (Jessop, 2022).

This paper situates in a different ontological starting point. Rather than conceiving markets as sites of accumulation structured by institutions (Pérez-Caldentey and Vernengo, 2023), it portraits them as socially constructed and fetishized arenas where capitalists and workers alike are subordinated to the the impersonal domination of capital social relations taking place *"at their backs"* (Vidal and Peck, 2012; Starosta, 2019; Basu, 2022). From this angle, both Neo-Kaleckian and Sraffian frameworks are here considered as macroeconomic models that underplay the intrinsic instability of the capitalist mode of production and its inner tendencies to stagnation and crisis.

This chapter develops a formal framework which leverages on the backbone of the SSM as a long-period theory of effective demand, but reinterprets it through a Marxist intuition: the long-run persistency of technological imbalances as structural bottlenecks for smooth reproduction of the capitalist mode of production. This insight is operationalized in the concept of unbalanced growth path, i.e in the long-run productive capacities are not scaled one to one as assumed by the SSM. Consequently, this article demonstrates that stagnation and crisis become a normal condition of capitalist development. Drawing on insights from the growth-models literature in CPE, it treats demand regimes not as exogenous backdrops but as unstable configurations centering the analysis in accumulation demand: expenditures that sustain the reproduction of capital under historically specific technological and institutional conditions, identified as *accumulation regimes*.

The remainder of the chapter is organized as follows. Section 2 revisits the literature on growth models, capacity utilization, and competition, clarifying the conceptual stakes of adopting a Marxist reading of markets and demand within a demand-led framework. Section 3 presents the analytical model: it lays out the demand typology, derives the standard Sraffian Supermultiplier, introduces the Harrod–Okishio investment function and the endogenous choice of technique to distribution, and defines the long-run behaviour of distributive conflict and accumulation demand. Section 4 extends the framework to what I call the Marxist Supermultiplier (MSM), establishes the conditions for unbalanced growth, stagnation, and crisis, and introduces a taxonomy of accumulation-regime crises. Section 5 concludes with implications for macroeconomic policy, comparative political economy, and future empirical research on accumulation regimes in both core and peripheral economies.

2 Literature Review

2.1 Keynesian Aggregate Demand and Policy-Relevant Multipliers

Start from the open-economy income identity:

$$Y = C + I + G + X - M, \tag{2.1}$$

with induced components $C = (1 - s)Y$, and imports $M = mY$, where s is the marignal propensity of savings and m is the marginal propensity to import. Defining exogenous demand components as $z \equiv I + G + X$, so z excludes imports by construction. Rearranging the goods-market equilibrium yields:

$$Y = \frac{z}{s + m - \phi}. \tag{2.2}$$

Two immediate Keynesian policy multipliers follow:

$$\frac{\partial Y}{\partial G} = \frac{1}{s+m}, \tag{2.3}$$

$$\frac{\partial Y}{\partial X} = \frac{1}{s+m}. \tag{2.4}$$

These expressions motivate counter-cyclical policy, exogenous changes in government expenditure can counteract downswings or upswings of investments and exports aiming to avoid idle economic resources, either of labour or capital utilization (Keynes, 1936).

2.2 The Sraffian Super Multiplier as Keynesian–Classical Synthesis

The Sraffian Super Multiplier (SSM) fuses Keynesian effective demand with the Classical surplus approach by abandoning price-clearing closures and adopting Sraffa (1960)'s input–output determination of relative prices, anchoring distribution in production relations while retaining Keynes (1936)'s principle that activity is demand-determined (Garegnani, 1978, 1979, 1992; Serrano, 1995). A common surplus-approach feature is the wage–profit frontier, which can be represented:

$$r\mu\, B\,(1-\omega) \tag{2.5}$$

where r is the profit rate, $\omega = \frac{w}{A}$, $\mu = Y/Y^p$ is the degree of capacity utilization, A labor productivity, and B capital productivity at normal level, i.e. is taken at level of productive capacities, but can be pumped up by demand or be idle depending the dynamics of capacity utilization μ. As argued by Tavani and Zamparelli (2018) is a standard distributive schedule across different traditions within surplus approaches.

Now, following the same standard keynesian model as presented above, but introducing the equation of motion of the investment share and a equation of motion of adjustment of capacity utilization, the SSM represent the short run with a gradual accelerator for the investment share responsive to utilization in deviation to its normal or planned degree, allowing spare capacities as a strategy of deterrence or stocking pile of inventories to face demand uncertainties (Ciccone, 1986):μ_n is determined exogenously.

$$\hat{\phi} \;=\; \gamma\,(\mu-\mu_n), \qquad \gamma>0 \tag{2.6}$$

Introducing the behavior of the investment share into the Keynesian aggregate demand equation, allows to identify output growth in the short-run:

$$\hat{y} \;=\; \hat{z} \;+\; \frac{\hat{\phi}}{s+m-\phi}. \tag{2.7}$$

Such that (2.7), allows to distinct the short from the long-run as driven by *endogenous* demand forces operating through the induced-investment channel ($\hat{\phi}$), which has proportional effects to the $\mathcal{SSM} = \dfrac{1}{s+m-\phi}$ [1].

Capacity utilization grows as the difference of gross output and capital accumulation, while the normal degree of capacity utilization is exogenously determined μ_n, by following the dynamic adjustment of capacity utilization as follows:

$$\hat{\mu} = g_Y - g_K \tag{2.8}$$

In the long-run, capital stock grows in line with demand by converging towards the normal rate of capacity utilization, which can be stated from the capital accumulation law of motion:

$$g_K = \phi B\mu - \delta \tag{2.9}$$

In the steady-state equilibrium long-run growth is defined by $\mu = \mu_n$, and capital accumulation becomes defined as:

$$g_Y = g_K = g_Z \tag{2.10}$$

Solving for ϕ^*:

$$\phi^* = \frac{g_Z + \delta}{B\mu_n} \tag{2.11}$$

Which is the required investment share of output that guarantees capacity grows at the same

[1] Where the model the every component of s,m,ϕ can be computed as a dynamic multiplier from observable data of national accounts

rate as demand while maintaining for a fully adjusted position $\mu = \mu_n$ (Vianello, 1985), however this convergence might be slow due slugish investment and cost of adjustment, or a gravitational center for capacity utilization (Ciccone, 1986).

Assuming induced consumption and imports as in the standard keynesian model, we have $S = sY$, $M = mY$, so total leakages are:

$$S + M = (s + m)Y. \tag{2.12}$$

The correct condition for equilibrium in the presence of autonomous expenditures is:

$$S + M = I + Z, \tag{2.13}$$

dividing both sides by Y:

$$s + m = \frac{I + Z}{Y}. \tag{2.14}$$

This implies that the average propensity to save and import must adjust to validate effective demand. Define the average saving-import ratio as:

$$s_a = \frac{I + Z}{Y} = \phi + \frac{Z}{Y}, \tag{2.15}$$

where $\phi = \frac{I}{Y}$ is the investment share of output.

Given that s and m are assumed as fixed parameters, the model closes via the endogenous adjustment of ϕ. Define the fraction:

$$f = \frac{s + m}{s_a} = \frac{s + m}{\phi + \frac{Z}{Y}}, \tag{2.16}$$

which governs how the average leakages adjust to match investment and autonomous demand without requiring changes in distribution.

At steady-state, where $\frac{Z}{Y}$ is constant, we can rewrite:

$$\phi = s_a - \frac{Z}{Y}, \tag{2.17}$$

ensuring that long-run consistency between investment, saving, and autonomous demand is maintained via the dynamic adjustment of the investment share.

Thus, while the model allows keeping the marginal propensity to save (consume) constante, the average propensity to save is not. Instead, it adjusts *endogenously* in the long-run via changes in the investment share ϕ, which responds to the dynamics of demand and capacity utilization, which in turn are a political problem in terms of how fiscal policy is defined contingently to historical balance of forces, while exports represents the integration of the national economy to the world market. This is the key mechanism by which the Sraffian Supermultiplier achieves model closure: without requiring changes in income distribution, neither long-run adjustment of the normal utilization rate as in the neo-kaleckian model of growth and distribution (Nikiforos, 2016b).

In a nutshell, the SSM lets the classical surplus approach as formalized by Sraffa (1960) coexist with a demand-led mechanism: relative prices and the wage–profit schedule remain anchored in production relations in the line of Smith, Ricardo, and Marx (Garegnani, 1984), while autonomous components (public spending, exports, some credit-driven items) set the pace of activity and investment adjusts so that capacity catches up and utilization gravitates to normal. Long-run growth tracks the trend of autonomous demand rather than shifts in income shares; distribution is treated as a historically contingent settlement, not the adjustment margin. Short-run multipliers fit naturally within this picture, and closure lies in the demand–investment–average-saving link, preserving the surplus logic without requiring permanent utilization gaps or distributive drift.

2.3 A Critical Assessment on the SSM toolkit for Comparative Political Economy

The SSM is an attractive framework for macroeconomics, but paritcularly for the study of comparative political economy. It shifts the ultimate driver of long–run growth to historically contingent, politically determined "autonomous demand" z, preserves Classical relative prices determination discarding atomistic decision making of aggregated individuals and firms, advancing an undertanding of markets as sites of accumultions embedded of relations of production. Furthermore, is consistent with the principle of distribution being conflictive, such that the understanding of Political Economy is not putting politics and economics in a toe bag, but understanding them as a unit. Just like the classics!

Nevertheless, the SSM model remains to have theoretical and empirical shortcomings. For example, Skott (2017) challenges the size and notion of autonomous expenditure arguing that autonomous expenditures is weak at empirically plausible shares of output to be a long-run stabilizing force. Furthermore, the candidate components proposed in the literature such as exports, government spending, but particularly consumption categories, and residential investments, are not compelling as trully autonomous components of aggregate demand. Rather, their dynamics in practice show strong signals of being endogenous to other macroeconomic drivers credit. The very features that render z politically meaningful also makes it historically endogenous, undermining the claim that an exogenous g_z can generically anchor the long run.

The internal mechanics of the model closure also requires stringing assumptions, as noticed by Nikiforos (2018), it requires investment to be tangent to perfectly elastic at the normal rate of capacity utilization. Otherwise, utilization gaps not necessarily close and Harrodian instability re-emerges. More broadly, the stability of the model relies in some ad-hoc assumptions, as for example, distinct normal capital productivity from normal capacity utilization. If these are lifted, as will be demonstrated in my theoretical framework below, stability does not hold.

Finally, Nikiforos et al. (2024) empirically the SSM model showcases empirical short-comings relative to its own predictions: its main feature is that autonomus demand components drive passive long–run investments, such that utilization rates leads the investment share ϕ in a dynamic counter–clockwise cycle convergence towards a fixed point in the (μ, ϕ) plane. The author empirical exercise present evidence for the U.S. post-war period where ϕ leads μ in clockwise cycles consistent with investment dynamics driving activity, and undermining the dynamic features of the model.

Nevertheless, the SSM have advanced an interesting literature reconciling the re-construction of classical political economy with Keynes original formulation of effective demand principle. Under my assesment, more importantly have presented a critique to neo-classical growth models in terms of the endogeneity of savings among other desirable features. Overall, its backbone as a demand-led model remains compelling. In the following section my theoretical framework hinges on its backbone to presente a marxist reformulation that reconciles the SSM with Harrodian Instability, and more importantly it overcomes a notion of autonomous demand providing an understanding of socially constructed framework for political economy, such that production relations are constitutive of markets and institutions without a divide between them (Vidal and Peck, 2012).

3 Theoretical Framework

3.1 Production, Capital Stock, and Endogenous Productive Capacities

The departing feature of my theoretical framework, is to portrait production with a Leontief production function:

$$Y = \min\{AL,\ \mu BK\}, \tag{3.1}$$

By doing so, there is no well-defined marginal rate of technical substitution. This rules out instantaneous factor substitution and the model remains immune to the Cambridge critique (Felipe and McCombie, 2014), which is a common feature of surplus approaches across contested approaches of heterodox macroeconomics (Tavani and Zamparelli, 2018). Furthermore, only capital productivity is directly defined by the capacity utilization rate, hence, both variables are demand-led following Keynes (1936) principle of effective demand. Capacity utilization is u. Labour productivity is A. Capital productivity at the *normal* level is $B = Y^p/K$; at effective demand, $\rho = Y/K = uB$. A note should be made here that in distinction to other surplus approaches where for methodological purposes the normal rate of capacity utilization is purposley conflated with the one at full capacity utilization, i.e. $\mu = 1$ if $Y = Y^p$.

Furtheremore, a Non-Accelerating Inflation Rate of Unemployment (NAIRU) is not defined at specific employment level, which precludes a inflation unemployment phillip curve[2]. Nevertheless, a *normal* capacity utilization rate $\mu = 1$ has a corresponding employment rate $\hat{\eta} = 0$, which means that the rate of growth of employment equals the rate of growth of the labour force $\hat{l} = \hat{n}$, i.e. full employment.

The mechanization ratio is defined as $Q = A/B$, so $B = A/Q$. In growth rates, capital productivity at normal level can be expressed as $\hat{b} = \hat{a} - \hat{q}$. The law of transformation from capital accumulation to productive capacities is defined as:

$$Y^p = K^{\theta(t)} \quad \Rightarrow \quad \hat{y}^p = \dot{\theta}\,\ln K + \theta\,\hat{k}.$$

[2] Which is consistent with the original formulation of Phillips who identify the relation of unemployment with wages, not with inflation (Shaikh, 2016)

In the short run we set $\dot{\theta} = 0$, yielding the benchmark

$$\hat{y}^p = \theta\, \hat{k}. \tag{3.2}$$

Hnce, the growth rate of productive capacities is demand-led — by accumulation demand,and θ is a quasi-fixed parameter representing the elasticity of transforming capital accumulation into growth of productive capacities. [3] Implicitly, this also represents an elasticity of labour productivity growth in response to induced mechanization in terms of growth rates. Accordingly, the next subsection discuss why θ is endogenous to distribution considering the capitalist firm mechanization decision, showing how distribution and regime-state Λ shape the endogenous relation $\theta(\omega \mid \Lambda)$.

Notice that at full capacity utilization $\mu = 1$, without instantaneous substitution between productive factors in distinction to neo-classical theory, the choice of technique of a pair (A, B) arise from a profit-oriented, market-mediated decision-making, which might be portrait with an optimal benchmark despite capitalists firms are not necessarily decision-making efficient. In this setting, working-class agency to capture gains of technological change are essential to define if a new choice of technique undermine or strengthen profitability (Basu, 2019, 2022).

[3]Establishing a law of accumulation demand transfomation into productive capacities, allows to express the rate of growth of capital productivity in linear terms as follows:

$$\begin{aligned}
Y^p &= uBK \\
Y^p &= u\frac{A}{M}K \\
\hat{y}^p &= \hat{b} + \hat{k}
\end{aligned}$$

Hence,

$$\begin{aligned}
\hat{b} &= \hat{a} - \hat{q} \\
\hat{b} &= (\frac{\hat{a}}{\hat{q}} - 1)\hat{q} \\
\hat{b} &= (\theta - 1)(\hat{k} - \hat{l}) \quad \text{with} \quad e = L/N \quad \hat{e} = 0 \implies \hat{l} = \hat{n} \\
\hat{b} &= (1 - \theta)\hat{n} + (\theta - 1)\hat{k} \\
\hat{y}^p &= (1 - \theta)\hat{n} + \theta\hat{k}
\end{aligned}$$

3.2 Aggregate Demand

Aggregate demand is the channel through which production relations express themselves in the short run and constrain accumulation in the long run. Rather than decomposing demand into "autonomous" and "induced" components—as in multiplier models—here demand components are classified by the mechanisms through which they affect productive capacity and income distribution.

Two dimensions are relevant: (i) whether the component generates new capacity or merely sustains utilization, and (ii) whether it is endogenously driven by distributional conflict or determined by institutional or external conditions. Considering this dimensions, I present an analytical classification is summarized in the Table 1 where each component of aggregate demand is classified in terms of its capacity/distributive induction features while also the analytical treatment given hereafter.

Table 1: Aggregate Demand Components by Induction and Capacity Effects

	Capacity / Induction features	**Share**	**Analytically**
I	Capacity-creating and fully induced by accumulation and profitability expectations;	$\phi \equiv I/Y$	Accumulation demand
C	Non-capacity-creating, induced by distribution; enters the multiplier via c (or $s = 1 - c$).	$c \equiv C/Y$	Induced non-accumulation demand
M	Non-capacity-creating leakage; induced by distribution and relative prices. In open economy, subtracts in $Y = C + I + G + X - M$.	$m \equiv M/Y$	Induced non-accumulation demand
G	Policy-determined. Can be capacity-creating when directed to public infrastructure; may also be distributionally induced via forms of governmentality (Jessop, 2007). Baseline identification treats G as analytically exogenous.	G/Y	Non-induced, non-accumulation demand.
X	External demand. Treated as analytically exogenous, though in practice partly semi-induced by inter-sectoral distribution via exchange rate and policy management but still shaped by external conditions.	X/Y	Non-induced, non-accumulation demand.

Consumption and imports are distribution-induced and non-capacity-generating; investment is capacity-creating and linked to profitability and accumulation; government expenditure and exports are treated as non-induced in the baseline specificationthat sustain demand without expanding capacity.[4], representing fiscal policy and external conditions in order to distinct methodologically the role of the state and the insertion in the world-market as suggested by Marx (1971).

3.3 Consumption Demand and Endogenous Savings

In this framework, consumption is not a behavioral function of income but a structural outcome of distributional conflict. The composition of aggregate demand reflects how the social surplus is divided and how much of it is re-spent rather than accumulated. Formally, the share of consumption in output can be written as a class-weighted function of the wage share:

$$c(\omega) = c_\omega \omega + c_\pi (1 - \omega), \tag{3.3}$$

where c_ω and c_π denote the propensities to consume out of wages and profits, respectively. Since workers' consumption tends to exhaust income while capitalists save part of theirs ($c_\omega > c_\pi$), the aggregate saving rate follows directly as

$$s(\omega) = 1 - c(\omega) = (1 - c_\pi) - (c_\omega - c_\pi)\omega. \tag{3.4}$$

This formulation deliberately excludes any notion of "autonomous consumption." Following Skott (2012), there is little empirical or theoretical basis to treat consumption as independent of distribution: even when households without disposable income sustain expenditures, such demand is financed elsewhere—through public transfers, informal self-provision, or credit—each of which ultimately reflects class and institutional relations.[5] Likewise, credit-financed consumption cannot be considered non-induced, since credit access and its distribution across classes depend on profitability and asset ownership.

Differential propensities c_ω and c_π are therefore historically contingent rather than constant parameters: shifts in income distribution can either amplify or dampen aggregate demand, as illustrated by the Fordist experience in Latin America, where elite consumption leaned heavily toward imported luxury goods despite high overall demand.[6]

[4]Feedbacks such as fiscal rules for G or exchange-rate effects on X can be introduced in extended versions but I abstract from them in this formulation.

[5]See Kay (2010) on Latin American debates over marginality and the reproduction of labour outside formal wage relations.

[6]Kay (2010) discusses these conspicous consumption patterns in relation to dependency and elite repro-

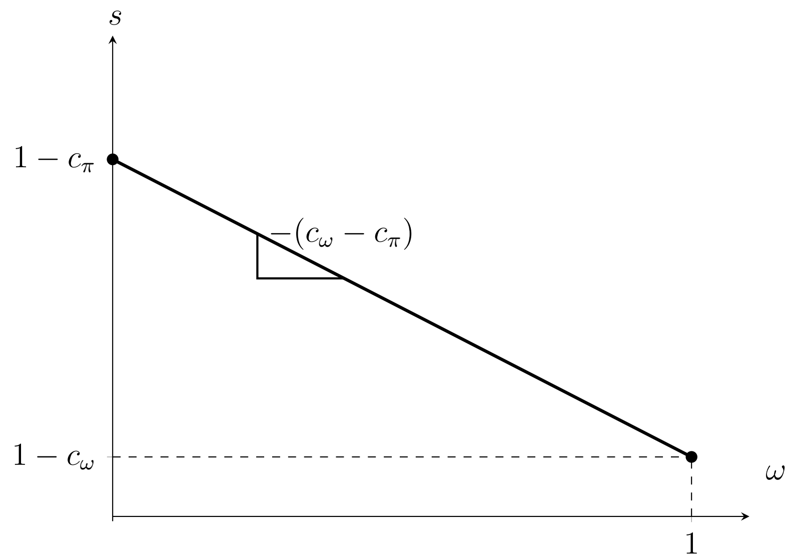

Figure 1: Endogenous saving rate $s(\omega)$ under class-differentiated consumption propensities. A higher wage share reduces the overall saving rate, reflecting that workers consume a larger portion of income ($c_\omega > c_\pi$). The slope $-(c_\omega - c_\pi)$ summarizes the distributive sensitivity of effective demand rather than a closure condition linking saving and investment.

Equation 3.4 thus expresses the classical premise that savings are endogenous to distribution, a key feature of the SSM but that here is treated as movements along the wage-profit frontier, not for any point of it. A higher wage share reduces the overall saving rate and vice versa. This relation does not rely on any equilibrium closure such as $I = S$, nor does it presume that savings are automatically reinvested. The surplus may be hoarded, consumed by elites, or diverted through the state, depending on institutional configuration and external constraints. What matters is that distribution governs the pace at which income is re-cycled into demand, linking accumulation to class behavior rather than to fixed propensities by assumption.

In the following section, this relation becomes the transmission mechanism through which technological imbalance (θ) and institutional adjustment (Λ) determine the evolution of accumulation and utilization.

3.4 Endogenous Imports and Distribution

Imports constitute both a leakage of demand and an expression of class and external relations. In an open economy, import demand reflects not only the technological structure of

duction.

production but also the distributive composition of expenditure. Class-based consumption implies that imports are endogenously linked to the wage share: workers and capitalists demand different bundles of goods, many of which have distinct import content. At the same time, part of imports correspond to intermediate and capital goods required for accumulation, so the external constraint evolves jointly with domestic distribution and investment.

Formally, a simple linear representation of the import propensity as a function of distribution is:

$$m(\omega) = m_\pi + \big(m_\omega - m_\pi\big)\omega, \tag{3.5}$$

where m_ω and m_π denote the marginal propensities to import out of wages and profits, respectively. Equation 3.5 captures the idea that a shift in income distribution alters the external leakage of demand. However, unlike the consumption and saving functions, it is difficult to establish an a-priori ordering of these parameters. In peripheral economies, the direction of the effect depends on regime-specific consumption patterns: an increase in the profit share may raise imports through luxury consumption and imported inputs, while a rise in the wage share may do so through higher demand for mass-consumption goods whose production depends on imported intermediates.

Latin American Structuralist and Dependency perspectives have long stressed that these patterns are not symmetric across classes or historical periods. Upper-income groups typically exhibit consumption baskets biased toward imported luxury goods, foreign services, and status consumption that mirrors elites in the core capitalist economies, while working-class demand may deepen import dependence through basic-goods deficits and food insecurity (Kay, 2010). Hence, the endogeneity of imports to distribution is itself regime-contingent and shaped by external hierarchy: profit-biased growth amplifies external leakages, whereas wage-biased growth exposes structural dependence on foreign production.

In this framework, imports therefore represent a transmission channel between internal distribution and the external constraint. They operate as a structural feedback: distribution affects import intensity, the trade balance influences accumulation capacity, and both jointly delimit the feasible path of growth. The external sector thus enters the model as an induced yet non-capacity-creating component of demand, linking domestic class dynamics to the world market without treating any part of demand as truly autonomous.

3.5 Okishio-Harrod Investment

Following Okishio (2022), investment I is determined by an accelerator function which grasp Okishio's understanding of Harrodian-Instability in terms of cumulative upward/downward causation. While this investment function does not include profitability explicitly as the Marxist tradition would feature , it implicitly does by representing the deviation of effective to normal profitability through the term $(\mu - 1)$. Here, is assumed as a convention that *normal capacity utilization* can be understood interchangeably with *full capacity utilization* understanding competition as anarchic. Thus, the normal level capacity utilization is identified at $\mu = 1$.

An Okishio-Harrod investment function is stated with a initial conditions of capital accumulation plus a an accelerator parameter scaled by the deviation of capacity utilization from normal capacity. Hence, if $u \neq 1$ Harrodian-Instability is triggered, accelerating the rate of growth of capital accumulation at increasing rates. So, we represent normalized investment per unit of capital stock as:

$$\frac{I}{K} = \frac{\Delta K}{K}$$

Which is equivalent to continuous time, considering $\dot{X}$ as the time derivative of X to:

$$\frac{I}{K} = \frac{\dot{K}}{K}$$

$$\hat{k} = k_0 + \gamma(\mu - 1), \quad \gamma > 0,$$

For the sake of simplicity, I assume the term $g_{K0} = 0$. Therefore, the Okishio-Harrod investment function becomes.

$$\hat{k} = \gamma(\mu - 1) \tag{3.6}$$

Where $u = Y/Y^p$ is the ratio of effective demand to productive capacities, hereafter, the *capacity utilization rate*, and is assumed that the rate of capital accumulation on the initial period is assumed in zero $k_0 = 0$ and also $\delta = 0$ depreciation for the sake of simplicity.

From an heterodox perspective, Okishio's understanding of the rate of capacity utilization with a convention of interchangeably use of full and its normal level to represent Harrodian-Instability, reflects the anarchy of the market. In the words of Matsuo (2025) who develops new microfundations for the Okishio-Harrod investment function, it represents the incapability of the market economy to remain in equilibrium, featuring instead an anarchic market dynamics in persistent disequilibrium.

3.6 Distribution, Choice of Technique $\theta(\omega \mid \Lambda)$ and Accumulation Regimes

In levels, the profit rate is defined as $r = \Pi/K$. Here, I follow Cajas Guijarro (2024) characterization of capitalists profit-oriented behavior aimed to maximize the effect of mechanization on *incremental* profitability. Around normal utilization and at given class-based distribution, the choice of technique is shaped by the effect of changes in mechanization ($\hat{q}$) to pump-up labour productivity ($\hat{a}$) as capitalist means for seeking profitability. Hence, capitalists optimal decision is given by a mechanization function at $\hat{e} = 0$, and correspondingly capacity utilization at normal level $u = 1$.

The transformation of capital to productive capacities θ is driven by mechanization effects, because the only component in aggregate demand capable build productive capacities is accumulation demand. Even though I acknowledge Okun Law effects on labour productivity, i.e. $\frac{\partial \hat{a}}{\partial \hat{\eta}} > 0$, these are cyclical and does not reflect the conditions of production at normal level (Jeon and Vernengo, 2008; Nabar-Bhaduri and Vernengo, 2024). Hence, abstracting them considering to identify normal conditions as a benchmark is a appropriate methodological abstraction for the long-run which does not entail that Say's law holds in this temporal horizon (Foley, 2013). Even though demand-led labour productivity effects might take place indirectly non-cyclical effects through distribution (ω) or technological regime state (Λ), this issue beyond the scope of this article. Overall, this formulation is consistent with a large array of heterodox economics literature once considered the parameter space of $\frac{\partial \hat{a}}{\partial \hat{q}}$ is state-dependent on Λ, which requires its own definition.

Definition 3.1 (Institutional–Technological State Λ)**.** Λ denotes the socially constructed configuration of institutions (social relations) and technology (productive forces) in which behavioral relations unfold. In what follows, Λ is the state under which the choice of technique is made. Formally, it might be represented by the parametric space that allows to identify the the mapping in which distribution and technique choice $\theta(\omega \mid \Lambda)$ is defined. Throughout this paper I will Λ as given and plausibly undergo to qualitative changes but without further specifications.

Already defined the state-dependent conditions under which capitalists takes place, we can proceed to present a formal behavioral definition of profit-oriented mechanization in the next definition.

Definition 3.2 (Profit-oriented mechanization behavior)**.** Define the the rate of profit in levels as:

$$r = \frac{\Pi}{K},$$

an accounting identity measuring the return on capital. Following Cajas Guijarro (2024), I assume that efficient capitalists profit-oriented behavior is given by aiming to maximize incremental profitability ($\hat{r}$) at normal capacity utilization ($u = 1$) and a given distribution (ω), by adjusting the pace of mechanization $\hat{q}$ to enhance labour productivity. Hence, maximization problem of capitalists is given by:

$$\max_{\hat{q}} \quad \hat{r} \;=\; \hat{a} - (1-\omega)\hat{q} \quad \text{s.t.} \quad \hat{a} = g(\hat{q} \mid \Lambda).$$

Where g is a differentiable function (so, continuous), strictly increasing in $\hat{q}$ ($g_{\hat{q}} > 0$), and featured by a regime-dependent curvature: standard standard diminishing returns $g_{\hat{q}\hat{q}} < 0$, although economies of scale or learning effects a given Λ state might arise locally with $g_{\hat{q}\hat{q}} > 0$.

Its worth noticing that Marx's understanding of capitalist competition as an anarchic process arises from the profit-oriented capitalist behavior and direct control of the labour process, here portrayed as the restriction of the optimization problem. While the optimal decision of the degree of mechanization for capitalists should be given at normal level, Okun effects on labour productivity might lead capitalist to overshoot/undershoot a perceived optimal degree of mechanization making their profit-oriented decisions inefficient. Despite stock of inventory management, logistics and data-driven decision-making processes have reduced inefficiencies in contemporary capitalism, still, as argued both by post-keynesian and marxist scholarship, firm decision-making has bounded rationality (Lavoie, 2014). Furthermore, its inner governance is given by a multi-scalar managerial structures featured by intrinsic conflicts leading managers to inefficient decision-making (Vidal, 2022). Therefore, capitalists are incapable to make decisions fully abstracting themselves from the business cycle. Rather, their expectations are driven by it, speculating on booms and busts by looking at market conditions where the dynamics of social relations are obscured, i.e. their behavior is governed by *fetishized accumulation arenas.*

Despite bounded rationality and capitalists herd behavior, and how the business cycle shapes their decision-making, profit-oriented behavior here is considered as a benchmark representing long-run aggregate behavior. At the methodological level, this abstraction allows us to characterize the choice of technique by isolating the condition that links distribution to mechanization. In particular, the first-order condition of the optimization problem program in Assumption Proposition 3.2 yields the following lemma.

Lemma 3.3 (Optimal mechanization: problem and FOC)**.** *Under Assumption* **??***, the optimal mechanization in normal conditions of operation for the capitalist firm is a result of the following program:*

$$\max_{\hat{q}} \ \hat{r} \ = \ \hat{a} - (1-\omega)\hat{q} \quad \textit{subject to} \quad \hat{a} = g(\hat{q}, \omega \mid \Lambda).$$

Any interior solution $\hat{q}^*(\omega \mid \Lambda)$ *satisfies*

$$g_{\hat{q}}(\hat{q}^*(\omega \mid \Lambda), \omega \mid \Lambda) = 1 - \omega.$$

Proof (reduced form; full derivation in Appendix A). Substitute the constraint to obtain an unconstrained maximization problem with respect to $\hat{q}$ yielding the first order condition $g(\hat{q}, \omega \mid \Lambda) = (1-\omega)\hat{q}$. See derivation and algebra arriving to equation (A.1). ■

The condition from lemma 3.3 states that induced mechanization adjusts until the marginal productivity gain from $\hat{q}$ is exactly offset by its distributive cost $(1-\omega)$, conditional on the state representation of technological and institutional conditions (Λ) defined outside the system.

From lemma 3.3 we can state the following definition:

Definition 3.4 (Endogenous choice of technique)**.** The capacity-transformation elasticity induced by the optimal technique is:

$$\theta(\omega \mid \Lambda) \;=\; \frac{g(\hat{q}^*(\omega \mid \Lambda))}{\hat{q}^*(\omega \mid \Lambda)}$$

As previously stated, θ measures the elasticity of capacity to capital accumulation at normal level.

Lemma 3.5 (Re-Distribution effect on the Choice of Technique)**.** *Holding* Λ *fixed and assuming* $g_{\hat{q}\hat{q}}(\hat{q}^*) \neq 0$

$$\frac{d\theta}{d\omega} = \frac{(1-\omega)-\theta}{g_{\hat{q}\hat{q}}(\hat{q}^*)\,\hat{q}^*}$$

Proof (reduced form; full derivation in Appendix A). From the total derivative of θ, we get the general form of $\dfrac{d\theta}{d\omega}$ at equation (A.9):

$$\frac{d\theta(\omega \mid \Lambda)}{d\omega} = -\,\frac{\big[(1-\omega)-\theta(\omega \mid \Lambda)\big]\,(1+g_{\hat{q}\omega}(\hat{q}^*,\omega \mid \Lambda))}{g_{\hat{q}\hat{q}}(\hat{q}^*,\omega \mid \Lambda)\,\hat{q}^*(\omega \mid \Lambda)} \;+\; \frac{g_{\omega}(\hat{q}^*,\omega \mid \Lambda)}{\hat{q}^*(\omega \mid \Lambda)}.$$

Assuming no direct effect from distribution on the mechanization function ($g_\omega = 0$) neither a cross indirect effect of distribution through mechanization ($g_{\hat{q}\omega} = 0$), such that distribution shapes mechanization solely by technological conditions expressed in the curvature $g_{\hat{q}\hat{q}}$. Hence, the benchmark effect of distribution on the choice of technique is given by equation (A.10):

$$\frac{d\theta(\omega \mid \Lambda)}{d\omega} = -\,\frac{(1-\omega)-\theta(\omega \mid \Lambda)}{g_{\hat{q}\hat{q}}(\hat{q}^* \mid \Lambda)\,\hat{q}^*(\omega \mid \Lambda)}$$

■

Thus, the gap $(1-\omega)-\theta$ represents the marginal payoff to mechanization (incremental profit share), relative to the average productivity gain from mechanization. If the response to mechanization is concave ($g_{\hat{q}\hat{q}} < 0$ -— a standard diminishing-returns regime, the sign of $d\theta/d\omega$ equals the sign of $(1-\omega)-\theta$, meaning that becomes a critical threshold depending the value of θ. When $1-\omega > \theta$ (profit share larger than the average productivity gain), an re-distribution favourable to the working class (i.e. a fall in the profit share) raises θ, likewise if $1-\omega < \theta$, an increase in ω lowers θ. Nevertheless, if the productivity response is convex due economies of scale or learning effects of mechanization $g_{\hat{q}\hat{q}} > 0$, the sign of the interpretation reverse, such that depending Λ state the effect of distribution on θ might flip. For this reason, $g_{\hat{q}\hat{q}}$ is the channel that generates the regime-switching behavior of $\theta(\omega)$.

3.7 Class Conflict and Accumulation Regimes

Already defined the endogenous choice of technique to distribution at given institutional and technological landscape, the toolkit to provide a definition of an accumulation regime which is necessary to layout the argument that in contrast to the SSM its marxist variant posits that the long-run is featured by structural crisis. This argument requires two additional definitions: i) the intrinsict instability of class conflict in the long-run, and ii) the socially constructed configuration of an accumulation regime.

Definition 3.6. Define the dynamics of distributive conflict *in the long-run* as *"Intrinsically Instable Class-Struggle"* (IICS) as follows. Given an institutional–technological state Λ, suppose the wage–share growth in the long-run is featured by a feedback mechanism such that the class gaining traction in the distributive conflict, leverage from more power given by a set of institutional and technological conditions Λ, formally:

$$\hat{\omega} = \Omega(\omega \mid \Lambda), \qquad \left.\frac{\partial \Omega}{\partial \omega}\right|_{\Lambda} > 0.$$

This means that at a given Λ, higher (lower) ω tends to raise (lower) $\hat{\omega}$, hence class power is self-reinforcing showcasing the polarizing tendencies of distributive conflict in the long-run. While this dynamical equation resembles Nikiforos (2016a) time-varying dynamics of growth and distribution, this formulation is actually grounded in terms of Kalecki (1943) argument on the political the political aspects of full employment. If unemployment shrinks in a trajectory towards full employment, working-class bargaining power becomes self-reinforcing with a ceiling at full employment requiring a unique set of institutions to be sustained as higher wage share induce political empowerment of the working class.

Yet, following Marx (1999) notion of the reserve army of labour, in conjunction with the introduction of capital-using and labour-saving technical change increasing tendency to expand the pool of unemployed (Marx, 2001), undermining working-class capacities to captures productivity gains further reinforcing capitalist class-power *vis-a-vis* shrinking labour power (Basu, 2022, 2024). Furthermore, the institutional and technological historical conditions represented by Λ, evolve with this tensions embedded in the state understood as a social relation itself with a clear tendency of rising power of intellectual over manual labour, shifting the balance of power between class fractions, reshaping the political arena on which distributive struggle unfold (Jessop, 2007). Altogether, these arguments provide a substantive foundation for the Proposition 3.6, emphasizing the polarizing and path-dependent character of the distributive conflict in the long-run.

Finally, given Proposition 3.2, Proposition 3.4 and Proposition 3.6 Proposition D.2 an Accumulation Regime $\mathcal{R}$ can be defined as follows:

Definition 3.7 (Accumulation Regime $\mathcal{R}$)**.** An accumulation regime can be defined as the finite sequence of variables and parameters:

$$\mathcal{R} \equiv \left\{ \theta(\cdot \mid \Lambda),\ \Omega(\cdot \mid \Lambda),\ \gamma,\ \hat{z},\ s(\omega),\ m(\omega),\ \Lambda \right\}$$

Such that $\mathcal{R}$ collects the capacity–accumulation mapping (θ), the long-run distributive drift (Ω), the accelerator parameter (γ), the growth rate of analytically exogenous demand ($\hat{z}$), endogenous savings and imports ($s(\omega), m(\omega)$), and the state Λ.

Reading guide. Holding $\mathcal{R}$ fixed, we study local dynamics in (μ, ϕ); changing Λ (and thus the behavioral relations) moves us across regimes. The IICS definition characterizes the long-run distributive drift component $\Omega(\cdot \mid \Lambda)$ within $\mathcal{R}$.

3.8 The Dynamic System of Accumulation Demand and Capacity Utilization

3.8.1 The Pace of Capacity Utilization

This section develops the dynamics of capacity utilization $\mu = Y/Y^p$ in a open economy with government expenditure and exports identified as non-induced, non-accumulation demand fulfilling and an Okishio–Harrod investment function.

Dynamically, the pace of capacity utilization is given by the identity:

$$\hat{\mu} = \hat{y} - \hat{y}^p \tag{3.7}$$

Thus, we can express it by recalling three core relations previously identified: i) the Okishio-

Harrod Investment Function (3.6), ii) the Sraffian Super Multiplier (**??**), iii) the interplay of the growth rates of effective demand and productive capacities driven by accumulation demand (3.2). This formulation has as a result imposing supply-side constraints to the system, but still being fully demand-led:

$$
\begin{aligned}
\hat{k} &= \gamma\,(\mu - 1) \quad \gamma > 0 \\
\hat{y} &= \hat{z} + \frac{\phi}{s - \phi + m}\hat{\phi} \\
\hat{\mu} &= (\hat{y} - \theta\hat{k}) \quad \theta > 0
\end{aligned}
$$

From these expressions is possible to derive the following reduced expression for the rate of growth of capacity utilization as a function of the Super Multiplier and the rate of growth of aggregate demand:

$$
\hat{\mu} = \underbrace{\hat{z}}_{\text{non-induced non-acc. demand}} + \underbrace{\frac{\phi}{s - \phi + m}}_{\text{Super Multiplier}}\hat{\phi} - \underbrace{\theta\gamma(\mu - 1)}_{\text{Harrodian Supply Expansion}}
$$

This expression reveals that *the growth rate of capacity utilization, $\hat{\mu}$, is determined by a combination of demand-side forces and supply-side feedback, which becomes the marxist reformulation of the Sraffian Super Multiplier.* As can be appreciated, a key feature of the model is that in this case Harrodian-Instability appears as a dampening effect mediated by θ to the rate of growth of capacity utilization. Thus, a reversal to instability emerges not only from within the Sraffian Super Multiplier, but also through the technological conditions in which capital accumulation is transformed into productive capacities, which as will be demonstrated later are contingent to the distributive conflict in a regime-switching pattern.

The pace of capacity utilization is identified by three components. First, this expression captures that in the long-run, there is proportional relation with the growth rate of the non-capacity-generating non-induced demand z.

Second, captures the endogenous response of investment to demand conditions as in the Sraffian Super Multiplier, were the relative shares of savings, investments, and imports define the amplification or dampening of the multiplier to the share of investment dynamics. It should be noticed that in the framework of the MSM, ϕ is not merely an accommodating variable as in the SSM. Rather, is driven by Harrodian-Instability which is key for the dynamic system to take non-linear forms and accumulation demand and the plausibility of crisis in the long-run. Notice that this setup requires that the super multiplier is well-defined and positive which holds if and only if the investment share ϕ is less than the total leakages to savings and imports $(s + m)$. Nevertheless, the instability of the super multiplier also would become a feature of endogenous instability of the system and cumulative causation towards over-accumualtion and crisis.

The interpretation is straightforward: in an economy where the investment share ϕ is small, the responsiveness of capacity utilization growth to changes in $\hat{\phi}$ is limited — any expansion in induced investment generates only a weak acceleration in demand growth. Conversely, as ϕ increases, the effect of changes in $\hat{\phi}$ on $\hat{\mu}$ grows stronger. Thus, the marginal effect of investment growth on capacity utilization is itself a function of the degree to which the economy is investment-led. This insight captures a fundamental dynamic of the MSM framework: even in a context driven by non-capacity-generating components of demand, the responsiveness of the system to induced accumulation becomes defined on the order of magnitude of the investment share and saving–import regime, which portrait structural features of a capitalist economy.

Finally, the term, $-\theta\gamma(\mu - 1)$, introduces a potentially *stabilizing supply-side effect* into the system. When utilization exceeds its normal level ($\mu > 1$), capital accumulation accelerates, expanding productive capacity and slowing the growth of utilization. Conversely, when $u < 1$, the effect turns expansionary. The strength of this feedback is governed by capitalist behavior in terms of their responsiveness of investment to utilization (γ) and the technological conditions determined on how efficient is the process of transformation from capital accumulation to productive capacities (θ).

This result has a parallel interpretation to the behavior of the investment share. A decrease in θ — understood as a decline in the efficiency of capital accumulation transformation into productive capacities, potentially due to labor-saving and capital-using technological change *à la* Marx — reduces the capacity of the system to expand potential output per unit of investment. In such a context, if the economy is in a state of over-utilization ($\mu > 1$), Harrodian Supply Expansion effect turns positive: that is, declining θ contributes to a rise in $\hat{\mu}$, reflecting the pressures in the system to expand output despite the technical constraints to expand productive capacities. Conversely, in a state of under-utilization ($\mu < 1$), the same decline in θ might lead to a fall in $\hat{\mu}$, reinforcing the contractionary dynamics as excess capacity and diminished capital productivity combine to weaken the incentive and capacity for investment-led expansion.

This behavior highlights the dual role of θ: not only mediates the supply-side transformation of investment into capacity, but it also dynamically feeds back into the effective utilization of productive capacity themselves. Its decline therefore amplifies demand-side pressures in the short run while structurally constrain capital accumulation in the long run.

In sum, this expression formalizes how *capacity utilization dynamics emerge from the interplay between demand growth and technical accumulation conditions*, with Harrodian Instability moderated — but not eliminated — by induced investment dynamics and productive capacity transformation driven by accumulation demand.

3.8.2 Investment Share Dynamics

In this section, I analyze the dynamic behavior of the investment share ($\phi = I/Y$) in the context of a Marxian modification of the Sraffian Supermultiplier. Our primary goal is to characterize how Harrodian instability from capacity utilization (u) can dynamically reverse within a demand-led framework driven by accumulation dynamics.

To understand the Marxian modification of the Sraffian Supermultiplier, we must analyze the dynamic behavior of the investment share, $\phi = I/Y$, as a key link between accumulation and demand. Capital accumulation causes demand through two distinct channels: (i) fluctuations in capacity utilization, which trigger Harrodian instability, and (ii) the expansion of productive capacity, which transforms the trajectory of the growth path, i.e. the Marxist Super Multiplier framework is embedded with hysteresis but not as a mere convention. *The central theoretical question becomes how Harrodian-Instability can be reversed from within a demand-led system driven by the profit-oriented process of capital accumulation itself?*.

We define the investment share in output as $\phi = \dfrac{I}{Y}$. Taking logs and differentiating with respect to time:

$$\hat{\phi} = \hat{I} - \hat{y}$$

Although we have expressions for the accumulation function from (3.6), (**??**), and (3.2), we stil lack an expression for $\hat{I}$. But actually, we are interested on the relation of accumulation dynamics and the super multiplier expressed in part within ϕ, i.e. the dynamics of the investment share which relates growth with capital accumulation, i.e. the pace of capacity utilization. To resolve this, I shall proceed departing from the following accounting identity:

$$\frac{I}{Y} = \frac{I}{K}\frac{K}{Y^p}\frac{Y^p}{Y}$$

Log differentiating we get:

$$\hat{\phi} = \hat{\hat{k}} + (\hat{k} - \hat{y}^p) - (\hat{y} - \hat{y}^p)$$

Using both expressions for the pace of capacity utilization rate (3.2), and (3.7), we can express previous identitiy as:

$$\hat{\phi} = \hat{\hat{k}} + (1 - \theta)\hat{k} - \hat{\mu} \tag{3.8}$$

Its worth to stop here and discuss each element of this expression. First, the term $\hat{\hat{k}}$ correspond to the second derivative of the capital stock's growth rate — i.e., the acceleration of the rate of accumulation itself. The economic intuition of this term, is thtat captures the momentum of the accumulation process. If the term is positive, meanining that capital accumulation is increasing at increasing rates (positive acceleration), reflects Okishio (2022) upward cumulative causation of capital accumulation driven by rising investment expectation of capitalist. Conversely, if $\hat{\hat{k}} < 0$ the pace of capital accumulation is decelerating, in other words, if $\hat{k} > 0$ firms are still investing, but less aggressively. On the other hand, if the case is that $\hat{k} < 0$ net investment would be negative meaning economically that productive capacities are being destroyed but this takes place at increasingly slower pace. In a nutshell, $\hat{\hat{k}}$ represents the timing of capital accumulation, weather the capitalist class is not only investing but speeding up or slowing down its investment drive.

Recalling the Okishio-Harrod accumulation function from equation (3.6), we can express the acceleration of capital accumulation $\hat{\hat{k}}$ as $\gamma\mu\hat{\mu}$.[7] This is a quite intuitive expression.

[7]From the Okishio-Harrod accumulation function, we can easily find the second derivative respecto to

If utilization is high and rising, investment expectations are not just positive — they are accelerating. Oppositely, if utilization is falling – even from a high level, accumulation decelerates. This captures the nonlinear feedback-loop inherent in demand-led accumulation dynamics that the system is capable to grasp.

Let's unpack the term $(1-\theta)\hat{k}$. Given our previously defined Harrodian investment function *à la* Okishio, $\hat{k} = \gamma(\mu - 1)$, and considering that θ represents the fraction of capital accumulation required to sustain the growth of potential output, the expression $1 - \theta$ represents a structural gap between actual capital accumulation and the technical conditions needed for balanced growth. Thus, if $1 - \theta > 0$, the economy becomes conditioned by an stagnation over-accumulation tendency, manifested as an excessive rise in the investment share. This scenario, resembles a stage of capitalist development prone to an overheating crisis were the excess of accumulation demand feeds positively the Super Multiplier effect up to a point that the glut of fixed capital formed by excessive investment becomes uselss, because accumulation demand increasingly dominates the composition of aggregate demand destabilizing a balanced growth trajectory. Conversely, if $1 - \theta < 0$, the capitalist economy becomes structurally conditioned by an stagnation tendency towards under-consumption of accumulation demand, i.e. characterized by insufficient growth of the investment share relative to the requirements to sustain the growth of productive capacities. This scenario, resembles depressive phases of capitalist development mostly known as secular stagnation, undermining aggregate demand expansion by weakening investment-driven dynamics. Consequently, the Super Multiplier shrinks effect becomes constrained, as inadequate investment dampens the system capabilities to achieve a growth trajectory capable to sustain productive capacities.

Beneath this tendencies towards over-accumulation or under-consumption of productive capacities, lies the contradiction of social relations at the core of the capitalist mode of production: the labour process. The measure of capital productivity under normal conditions $\frac{Y^p}{K}$, conceals capital's drive to induce higher degrees of labour productivity through choice

time as follows:

$$
\begin{aligned}
\hat{\hat{k}} &= \frac{d}{dt}\hat{k} = \gamma\frac{d}{dt}(\mu - 1) \\
&= \gamma\dot{u} \\
\text{and}& \\
\hat{\mu} &= \frac{\dot{u}}{u} \quad \Rightarrow \quad \dot{u} = u\hat{\mu} \\
\hat{\hat{k}} &= \gamma u\hat{\mu} \qquad (3.9)
\end{aligned}
$$

of techniques featured by higher degrees of mechanization. Thus, when labor productivity growth persistently exceeds or lags behind the pace at which new investments are converted into productive capacities, the conditions for capitalist crisis emerges. This tension reveals the underlying contradiction between the imperative of capital to continually transform labor productivity gains into accumulation, and the technical capacity to effectively realize this accumulation within the capitalist mode of production.

The result of the rate of growth of capacity utilization $\hat{\mu}$ being proportionally negative with the dynamics of the investment share could seem contradictory with the Harrod-Okishio investment function. Nevertheless, this might be interpreted in three ways. First, it expresses the limits of aggregate demand due possible lack of realization of commodity production. If utilization rises rapidly, output grows faster than productive capacities narrowing the space for profitable investments because existing capital is already being used at high intensity. Hence, firms delay investments not because market conditions are unfavorable, rather, because they are excessively good creating bottlenecks such as cost pressures, or uncertainty that future investments would be as attractive as current ones. Here lies a paradox of economic growth under the capitalist mode of production: it create its own limits, not because scarcity of capital, but from uncertainty from the realization of produced commodities. On the contrary, if utilization is declining the economy is moving towards under-utilization, i.e. effective demand slows relative to productive capacities. While this might seem a worse scenario for capitalist, it also opens up space for capital to be profitably reinvested, particularly if firms expect this path only to be temporary. In this case, firms would accelerate investment not because conditions are good, instead, because they are too loose, and market conditions might set an environment of cheap capital and abundant supply of inputs and labour prevention opportunities in markets that are too competitive.

Second, it represents an endogenous self-adjustment mechanism of the capitalist mode of production. Both over and under-utilization have opposite but linked effects on investment share dynamics. When capacity utilization is growing, the negative effect on the investment share rate of growth, reflects a constraint of induced hesitation. In other words, investment slows relative to growth to avoid overheating and capacity bottlenecks. While this endogenous self-adjustment built-in mechanism might work as a stabilizer of the system, it also can trigger oscillatory behavior prone to stagnation and crisis. On the other hand, if capacity utilization is decreasing the inverse logic gets into play, reflecting an anticipatory or counter-cyclical investment where firms might choose to modernize or reposition themselves under slack market conditions. In a nutshell, this bidirectional feedback mechanism captures both demand-led growth inertia and the contradictory features of accumulation demand under the capitalist mode of production.

Third, investment behavior in this setting is not mechanically proportional to capacity utilization. The sign of $\hat{\mu}$ conditions a qualitatively different logic of responses by capitalist enterprises, which might be labeled reactive restraint and strategic anticipation. If capacity utilization is growing, investment is reactive and cautions. Under this conditions firms might assess that better market conditions could be temporary, if firms behave with reactive restrain - many times they do not, investment might lag because firms need confirmation of sustained improved market conditions before committing to irreversible expenditure of capital outlays. This cautious backward-looking behavior might reduce $\hat{\phi}$ in the short run despite strong utilization. Conversely, a decreasing rate of growth of capacity utilization signaling worsening market conditions, capitalist enterprises might considered it an opportunity and behave with strategic anticipation investing preemptively while cost are low and resources supplies are good. Hence, the investment increases despite falling utilization, based on forward-looking expectations. This counter-cyclical response of investment is conditional on structural conditions, i.e. the pace of capital accumulation the structural tendencies towards over-accumulation or under-consumption of productive capacities. Overall, this interpretation considers that the formation of expectations by capitalist decisions are non-linear, investment might be cautions in booms but bold in periods of depressions, but only when structural conditions allows it. Intuitively, this captures a tension between capitalist risk aversion and strategic behavior shaping the asymmetric behavior of business cycles and possible structural breaks of accumulation dynamics.

Considering the growth rate of the investment share in equation (3.8), and the derivation of the expression (3.9) and substituting into the the investment share $\hat{\phi}$ expression at (3.8), we get an expression for the equation of motion of the investment share with two analytically distinct components:

$$\hat{\phi} = \underbrace{\hat{\mu}(\gamma\mu - 1)}_{\text{Demand-side feedback}} + \underbrace{(1-\theta)\gamma(\mu - 1)}_{\text{Supply-side structural effect}} \tag{3.10}$$

The first term, $\hat{\mu}(\gamma u - 1)$, captures the *demand-led dynamic feedback* from utilization to the investment share. When the rate of capacity utilization is increasing ($\hat{\mu} > 0$), this term measures the strength of firms' responses to rising demand, weighted by the sensitivity of investment to utilization (γ) and the level of utilization itself (u). If $\gamma u > 1$, then utilization growth amplifies the investment share — an expression of Harrodian instability. Conversely, if $\gamma\mu < 1$, investment reacts more cautiously, reflecting bounded rationality or uncertainty about the permanence of demand expansion.

The second term, $(1-\theta)\gamma(\mu-1)$, introduces a *structural supply-side effect* tied to the efficiency of capital accumulation. Here, θ denotes the productivity of capital, i.e. how effectively capital stock translates into productive capacity. If $\theta < 1$, capital accumulation is inefficient, and deviations from normal utilization ($u \neq 1$) reinforce changes in the investment share. In contrast, if $\theta > 1$, efficient transformation of capital into productive capacity can help neutralize demand shocks by easing potential supply constraints. This term embodies the feedback from the production side, modulated by both technological conditions and deviations from normal utilization.

Taken together, these two terms structure the behavior of the investment share in the MSM framework as the outcome of both dynamic responses to aggregate demand and deeper institutional-technical constraints on accumulation. The next section identifies the conditions under which Harrodian-Instability is reversed, based on the comparative statics of this decomposition.

4 Analytical Results: Stagnation, Partial Crisis and Structural Crisis

This section develops the analytical results of the Marxist Super Multiplier framework in a structured sequence of definitions and propositions, to provide a taxonomy that allows to differentiate stagnation tendencies, from crisis, and structural crisis. By doing so, under a behavioral definition of unstable dynamics of class-conflict in the long-run, a formal theorem arguing why in the long-run structural crisis of capitalism are unavoidable will be presented and proven. This analytical results under certain behavioral additions on the dynamics of class struggle shows the potential of the Marxist Super Multiplier as a versatile and robust theoretical framework which emphasize the role of historical conditions and institutional change to resolve structural crisis of capitalism. In this avenue, *accumulation regime* crisis are defined as a structural feature of the capitalism mode of production driven by the anarchic nature of market, here understood as *fetished accumulation arenas*, i.e. socially constructed institutions mystifying relations between individual and their social conditions as relations with things, instead of social relations.

4.1 Core Definitions and Basic Propositions

Definition 4.1 (Dynamic System of the Marxist Super Multiplier)**.** The dynamic system is defined by:

1. Variables: capacity utilization μ and the investment share ϕ.
2. Parameters: the marginal propensity to invest ($\gamma > 0$), the mechanization elasticity of labour productivity ($\theta > 0$), and the growth rate of analytically exogenous, non-capacity-generating, non-distribution-induced demand $\hat{z}$, where $Z = G + X$.
3. Laws of motion:

$$\hat{\phi} = \hat{\mu}(\gamma\mu - 1) + (1 - \theta)\gamma(\mu - 1), \tag{4.1}$$

$$\hat{} = \hat{z} + \frac{\phi}{s + m - \phi}\hat{\phi} - \theta\gamma(\mu - 1). \tag{4.2}$$

Proposition 4.2. *Consider the dynamic system on the admissible set* $\mathcal{A} = \{(\mu, \phi) : 0 < \phi < s + m\}$ *with parameters* $\gamma > 0$, $\theta > 0$, *and* $\hat{z}$. *Let*

$$\delta = \frac{\hat{z}}{\gamma\theta}, \qquad \mu^*_{\pm} = 1 \pm |\delta|.$$

Assume the nullcline discriminant $\Delta(\hat{z}) \geq 0$. *Then:*

1. Fixed points: *In the steady state the fixed points yields:* $\mu^*_{\pm} = 1 \pm |\delta|$, *with* μ^*_{+} *always feasible and* $\mu^*_{-} > 0$ *iff* $|\delta| < 1$.

2. Type: *On the admissible set, equilibria are nodes or saddles only.*

3. Unbalanced Growth Technology*: Define two types of unbalanced growth technological change* $\Theta_{over} := \{\theta > 1\}$ *and* $\Theta_{under} := \{0 < \theta < 1\}$. *Then:*

$$\theta > 1 \Rightarrow \ \mu^*_{-} \text{ is the saddle (Under-accumulation stagnation tendency)}$$
$$0 < \theta < 1 \Rightarrow \ \mu^*_{+} \text{ is the saddle (Over-accumulation stagnation tendency)}$$

4. Divergence around the saddle (case split). *Let* $\delta = \hat{z}/(\gamma\theta)$

   (a) Over-mechanization (OM) $0 < \theta < 1$ *(saddle* μ^*_{+}*):*

- *If* $\frac{1}{2} \leq \theta < 1$ *(early/shallower OM): no admissible cut on* μ_+^**; divergence is always on the* slow *side.*
- *If* $0 < \theta < \frac{1}{2}$ *(late/deeper OM): a zero-trace cut at* $|\delta| = \theta/(1-2\theta)$ *partitions* slow *divergence (*$|\delta|$ *below) from* fast *divergence (*$|\delta|$ *above).*

(b) Under-mechanization (UM): $\theta > 1$ *(saddle* μ_-^**, feasible if* $|\delta| < 1$*): there is a zero-trace cut at* $|\delta| = \theta/(2\theta-1)$*; divergence is* slow *for* $|\delta|$ *below and* fast *for* $|\delta|$ *above.*

Proof. All the dynamic analysis of system is collected at Section B, where all proofs for the proposition can be found. ∎

Proposition 4.3. *Let the* Harrodian slope *be expressed as a function of the state variables of the system defined on the admissible set* $\mathcal{A}$*:*

$$\Gamma(\mu, \phi; \theta, \gamma, \hat{z}) = \frac{d\hat{\phi}}{d\hat{\mu}}, \tag{4.3}$$

Hence,

1. Feasible signs: $\exists$ *admissible states such that* $\Gamma \begin{cases} > 0 & \textit{Harrodian Instability} \\ < 0 & \textit{Harrodian Reversal} \end{cases}$

Proof. See Section C which proof under the dynamic system Proposition 4.1, $\frac{d\hat{\phi}}{d\hat{\mu}}$ might be positive or negative in the short and the long-run. ∎

4.2 The Marxist Super Multiplier

Definition 4.4. Let $\Phi(\phi) = \dfrac{\phi}{s - \phi + m}$ be the Sraffian Super Multiplier on the admissible set $\{(\mu, \phi) : 0 < \phi < s + m\}$, and let the (local) Harrodian slope be $\Gamma(\mu, \phi; \theta, \gamma, \hat{z}) = \frac{d\hat{\phi}}{d\hat{\mu}}$. Defined except on a knife–edge set $\mathcal{K}$ where the derivative becomes undefined (See Section C). The *Marxist Super Multiplier* is defined as:

$$\mathrm{MSM}(\mu, \phi; \theta, \gamma, \hat{z}) = \Phi(\phi)\, \Gamma(\mu, \phi; \theta, \gamma, \hat{z})$$

Since $\Phi(\phi) > 0$ on $0 < \phi < s + m$, $\text{sgn}(\text{MSM}) = \text{sgn}(\Gamma)$. Unlike the standard Sraffian Super Multiplier, the MSM may be negative when the Harrodian slope is negative (reversal). This definition also holds for the long-run such that $\frac{d\hat{\phi}}{d\hat{\mu}}|_{\text{ss}} \neq 0$ (See Proposition C.2 in Section C).

Proposition 4.5. *Consider the dynamic system defined in Proposition 4.1, and Proposition 3.4 such that $\theta(\omega|\Lambda)$ is endogenous with $\theta \neq 1$. At given $\hat{z} \neq 0$, there is no interior steady state $(\hat{\mu}, \hat{\phi}) = (0, 0)$ with $u \in (0, \infty)$ and $\phi \in (0, s + m)$; in particular, no balanced-growth path exists. The only stationary cases are knife-edge: either (i) $\hat{z} \equiv 0$, or (ii) $\theta = 1$. Hence, introducing Harrodian-Instability the conclusions of the Sraffian Super Multiplier does not hold.*

Proof. A steady state requires $\hat{\mu} = \hat{\phi} = 0$. From the second equation,

$$0 = \hat{z} + \Phi(\phi)\hat{\phi} - \theta\gamma(\mu - 1) = \hat{z} - \theta\gamma(\mu - 1) \quad \Rightarrow \quad \mu - 1 = \frac{\hat{z}}{\theta\gamma}.$$

With $\hat{z} \neq 0$, we have $u \neq 1$.

From the first equation at steady state,

$$0 = (\gamma\mu - 1)\underbrace{\hat{\mu}}_{=0} + (1 - \theta)\gamma(\mu - 1) = (1 - \theta)\gamma(\mu - 1).$$

Since $\mu - 1 \neq 0$, we must have $\theta = 1$.

Thus, for $\hat{z} \neq 0$ a steady state exists only if $\theta = 1$. Under the MSM, $\theta = \theta(\omega \mid \Lambda)$ is endogenous and $\theta = 1$ becomes a knife-edge condition, which is generically violated by $\hat{z} \neq 0$. Therefore, no interior steady state exists.

If $\hat{z} = 0$, then $u = 1$ from the second equation and the first equation is also satisfied for any θ, yielding a non economically meaningful zero-growth stationary point. If instead $\theta = 1$, then the first equation imposes no restriction on u beyond $\hat{\mu} = 0$, and the second pins $\mu - 1 = \hat{z}/\theta\gamma$.

Therefore, with endogenous $\theta \neq 1$ and $\hat{z} \neq 0$, there is no balanced-growth steady state. By Proposition C.2, local slopes around fixed points induce Harrodian divergence or reversal rather than convergence if u^* is a saddle. Hence, in the long-run economic growth is not solely driven by analytically exogenous demand and *the conclusions of the Sraffian Super*

Multiplier does not hold in the long-run because Harrodian-Instability. ■

4.3 Class Conflict and Structural Crisis in the Long-Run

Definition 4.6. Let μ^*_{sad} denote the saddle branch selected by θ (see Proposition 4.2). In a neighborhood of μ^*_{sad}:

1. *Stagnation Tendency:* Structural feature of unbalanced growth, i.e. $\theta \neq 1$. By Proposition 4.2:

$$\begin{aligned} \theta > 1 \quad &\Rightarrow \quad \text{under-accumulation stagnation tendency} \\ 0 < \theta < 1 \quad &\Rightarrow \quad \text{over-accumulation stagnation tendency} \end{aligned}$$

   For $\theta > 1$, the saddle is μ^*_-. This regime is *stable despite over-heating*, i.e. that capacity-utilization above normal conditions might is stable, because over-heating is damped back thorugh formation of proudctive capacities, hence over-utilization of productive capacities might be sustained. However, the under-utilization side is prone to stagnation because instability rules in this state and divergence and capacity utilization might drifts around under-utilization of productive capacities. High growth is achievable by sustaining the system in over-utilization of productive capacities, which means ensuring that $\hat{z} > 0$, counteracting the export cycle if necessary through government expenditures.

   In the case of $0 < \theta < 1$, the saddle is μ^*_+, thus, stagnation tendency is reversed. The system is *stable in depressions*, i.e. under-utilization states can be easily contained, while *unstable during booms*, since over-utilization diverges (again subcritical if MSM $\leq$ 1). High growth paths are fragile and crisis-prone because the choice of technique embedded in θ is featured by an excessive glut of capital required for the reproduction of the system at given Λ.

2. *Partial Crisis:* A realized divergence with MSM > 1 that slows growth and cannot be reabsorbed under the given Λ, but can be resolved through a *nearby* institutional–technological shift $\Lambda \to \Lambda'$ within a similar parameter space. In this case, the adjustment offsets the unstable features of class conflict (see Proposition 3.6) so that $\theta(\omega \mid \Lambda')$ intersects a neighborhood with MSM ≤ 1, while the polarizing tendency of $\Omega(\omega \mid \Lambda')$ is not reversed. Thus resolution requires a modification of Λ that restores

feasibility without abolishing the underlying distributive dynamics.

3. *Structural Crisis:* Recall Proposition 3.7 an *Accumulation Regime* is defined as:

$$\mathcal{R} = (\theta(\omega \mid \Lambda),\, \Omega(\omega \mid \Lambda),\, \gamma,\, s(\omega),\, m(\omega),\, \Lambda).$$

A *structural crisis of capitalism* occurs when divergence with MSM > 1 cannot be resolved by any nearby Λ' within the same parameter space. Under the current regime, $\theta(\omega \mid \Lambda)$ does not intersect a neighborhood with MSM ≤ 1 near μ^*_{sad}, and the polarizing tendency of $\Omega(\omega \mid \Lambda)$ continues to reinforce instability. Resolution requires a qualitative transformation $\Lambda \to \Lambda'$ that *reverses* the unstable dynamics of Ω (or equivalently redefines the accumulation regime $\mathcal{R}$), so that the drift of ω is fundamentally redirected and MSM ≤ 1 becomes structurally attainable.

Proposition 4.7. *Let μ^*_{sad} be the saddle branch selected by θ, and define the utilization gap $g = u - u^*_{\text{sad}}$, the gap energy $E = \frac{1}{2}g^2$, and $\mathrm{MSM}^* = \mathrm{MSM}(\mu^*_{\text{sad}}, \phi^*)$. Then:*

(i) Structure ($S(\theta)$)*: The selection of the saddle at over/under-utilization of productive capacities μ^*_{sad} depends only on θ (Proposition 4.2), hence, it is independent of Harrodian feedback.*

(ii) Realization*: In any neighborhood of μ^*_{sad}, the system diverges (crisis) iff* $\mathrm{MSM}^* > 1$*, and it damps (no crisis) iff* $\mathrm{MSM}^* < 1$*. At* $\mathrm{MSM}^* = 1$ *the system is at a local threshold.*

Claim 4.7.1. Locally along the saddle, define *Crisis Energy* as:

$$\dot{E} \;=\; \underbrace{\frac{u^*_{\text{sad}}\,\theta\gamma}{(1-\mathrm{MSM}^*)^2}}_{K>0}\,(\mathrm{MSM}^*-1)\,g^2 \;+\; o(g^2).$$

Hence the sign of $\dot{E}$ is determined entirely by $\mathrm{MSM}^* - 1$.

Claim 4.7.2. Let $S(\theta) \in \{+,-\}$ denote the saddle branch and $\chi(\gamma, \theta, \hat{z}, \cdots) \in \{\text{crisis}, \text{no crisis}\}$ the realized outcome. Then

$$\chi \;=\; \mathbf{1}\{\mathrm{MSM}^* > 1\}.$$

Thus changes in θ affect χ only through the value of MSM^* (and a positive scale K), not through $S(\theta)$ itself. If two parameter vectors share the same MSM^* at their saddles, they yield the same realized classification, even if they select different branches.

Proof of Claim 4.7.1. Let $g = u - u^*_{\text{sad}}$, $E = \frac{1}{2}g^2$. Start from

$$\hat{\mu} = \hat{z} + \Phi(\phi)\,\hat{\phi} - \theta\gamma(\mu - 1).$$

At the saddle, $\hat{z} = \theta\gamma(\mu^* - 1)$ and $\hat{\phi}^* = 0$ under the nullclines. Subtracting the fixed-point identity and linearizing yields

$$(1 - \text{MSM}^*)\,\hat{\mu} = -\theta\gamma\,g + o(g).$$

Hence

$$\dot{g} = -\frac{u^*_{\text{sad}}\theta\gamma}{1 - \text{MSM}^*}\,g + o(g).$$

Multiplying by g gives the stated expression for $\dot{E}$. ■

Proof of Claim 4.7.2. From the previous display, $\dot{E} = K(\text{MSM}^* - 1)g^2 + o(g^2)$ with $K > 0$, so the sign of $\dot{E}$ is governed solely by $\text{MSM}^* - 1$. Therefore $\chi = \mathbf{1}\{\text{MSM}^* > 1\}$, independent of the branch label $S(\theta)$. ■

Proof of Proposition 4.7. Combine Claim 4.7.1 and Claim 4.7.2. ■

4.4 Structural Crisis of Capitalism in the Long Run

In the SSM, long-run growth is tied to analytically exogenous demand; but once Harrodian instability and unbalanced growth ($\theta \neq 1$) are admitted, the balanced-growth result fails (see Proposition 4.5). A long standing Marxist research program with a central concern on the entanglemnt between institutions and capitalist development (SSA, Regulation, Variegated Capitalism Approach) understanding the long-run as prone to stagnation, crisis and regime change. Recent work by Galanis et al. (2024) builds upon Arrighi (1994) and Arrighi and Silver (1999) formalizing how capital accumulation traverses stability, turbulence, leading to chaos. In the MSM this perspective is operational: when realized crisis persists under a given Λ; resolution requires a directional regime change from $\Lambda \to \Lambda'$ such that local dynamics is re-structured toweards stability if and only if the state Λ' is feasible. This reasoning can be stated as the following theorem on *Accumulation Regime Crisis.*

Theorem 4.8. *Consider the MSM dynamic system of Proposition 4.1 on the admissible set $\mathcal{A} = \{(\mu, \phi) : 0 < \phi < s + m\}$ with $\gamma > 0$, given $\hat{z}$, and θ endogenously determined by the* *choice of technique* $\theta(\omega \mid \Lambda)$ *as in Propositions 3.2 to 3.5. Assume IICS (Proposition 3.6) and feasibility* $\Delta(\hat{z}) \geq 0$ *(Section B). Let* μ^*_{sad} *be the saddle selected by* θ*, define the gap* $g = u - u^*_{\text{sad}}$*, the energy* $E = \frac{1}{2}g^2$*, and the instability margin*

$$\varrho(\omega \mid \Lambda) \;=\; \text{MSM}^*(\gamma, \theta(\omega \mid \Lambda), \hat{z}) \;-\; 1.$$

Then, in a neighborhood of μ^*_{sad} *off knife-edges:*

1. *Orthogonality (structure vs realization).* $\operatorname{sgn} \dot{E} = \operatorname{sgn} \varrho$*, hence divergence (crisis) iff* $\varrho > 0$*, damping iff* $\varrho < 0$*, threshold at* $\varrho = 0$*, independently of which branch is the saddle (Proposition 4.7).*

2. *Direction of crisis (boom vs slump). If* $\varrho > 0$*, then*

$$\begin{aligned} g > 0 \;\Rightarrow\; \dot{g} > 0 \text{ and } \hat{\mu} > 0 \quad &\textit{(boom-type crisis, over-utilization divergence)}, \\ g < 0 \;\Rightarrow\; \dot{g} < 0 \text{ and } \hat{\mu} < 0 \quad &\textit{(slump-type crisis, under-utilization divergence)}. \end{aligned}$$

   Equivalently, with the direction index $D = (\text{MSM}^* - 1)\, g = \varrho\, g$*, one has*

$$\varrho > 0 \quad \Rightarrow \quad \operatorname{sgn} D = \operatorname{sgn} g = \operatorname{sgn} \hat{\mu}.$$

3. *Persistence of structural crisis. If there exists an interval* I *such that* $\varrho(\omega \mid \Lambda) \geq \eta > 0$ *for all* $\omega \in I$ *and the IICS orbit* $\mathcal{O}(\omega_0 \mid \Lambda)$ *is forward invariant in* I*, then every visit to the saddle neighborhood yields realized crisis and no local reabsorption is possible under* Λ*.*

4. *Directional resolution via regime change. Let the* $\mathcal{L}_\Omega \varrho(\omega \mid \Lambda)$ *along the IICS trajectory be:*

$$\mathcal{L}_\Omega \varrho(\omega \mid \Lambda) \;=\; \frac{d}{dt}\varrho(\omega_t \mid \Lambda) \;=\; \frac{\partial \varrho}{\partial \theta}\frac{d\theta}{d\omega}(\omega \mid \Lambda)\, \Omega(\omega \mid \Lambda) \;+\; \frac{\partial \varrho}{\partial \omega}(\omega \mid \Lambda).$$

   - Necessity. *If* $\mathcal{L}_\Omega \varrho \geq 0$ *on* $\{\omega : \varrho > 0\}$*, the orbit cannot reach* $\{\varrho \leq 0\}$*, so crisis persists under* Λ*.*

   - Sufficiency with directional content. *If there exists* Λ' *such that on* $\{\omega : \varrho(\omega \mid \Lambda') > 0\}$*,*

$$\mathcal{L}_{\Omega'} \varrho(\omega \mid \Lambda') \;<\; 0,$$

     then the orbit hits $\{\varrho \leq 0\}$ *in finite time and subsequent visits to the saddle* *neighborhood are damped. In the benchmark $\partial\varrho/\partial\omega = 0$, the condition reduces to*

$$\frac{\partial\varrho}{\partial\theta}\frac{d\theta}{d\omega}(\omega \mid \Lambda')\,\Omega(\omega \mid \Lambda') \;<\; 0 \quad \text{on } \{\varrho > 0\}.$$

Hence resolution requires a regime change $\Lambda \to \Lambda'$ that redirects the IICS–technique push against the prevailing instability margin; the realized crisis direction (boom or slump) is given by the sign of g until ϱ crosses to ≤ 0.

Proof. By Proposition 4.7 there exists a local representation

$$(1 - \mathrm{MSM}^*)\,\hat{\mu} \;=\; -\theta\gamma\, g \;+\; o(g), \qquad \dot{g} \;=\; u^*_{\text{sad}}\,\hat{\mu} \;+\; o(g),$$

which yields

$$\dot{g} \;=\; -\frac{u^*_{\text{sad}}\theta\gamma}{1 - \mathrm{MSM}^*}\, g \;+\; o(g) \;=\; \alpha^*\, g \;+\; o(g), \quad \alpha^* \;=\; -\frac{u^*_{\text{sad}}\theta\gamma}{1 - \mathrm{MSM}^*}.$$

Part (1) follows from Claim 4.7.1: since $K > 0$,

$$\dot{E} \;=\; K\big(\mathrm{MSM}^* - 1\big)g^2 \;+\; o(g^2) \;=\; K\,\varrho\, g^2 \;+\; o(g^2),$$

so $\operatorname{sgn}\dot{E} = \operatorname{sgn}\varrho$.

For (2), when $\varrho > 0$ one has $\alpha^* > 0$, hence $g\,\dot{g} > 0$ for $|g|$ small; moreover $\dot{g} = u^*_{\text{sad}}\hat{\mu} + o(g)$ implies $\operatorname{sgn}\hat{\mu} = \operatorname{sgn}\dot{g}$ locally. Therefore $g > 0 \Rightarrow \dot{g} > 0, \hat{\mu} > 0$ (boom-type crisis) and $g < 0 \Rightarrow \dot{g} < 0, \hat{\mu} < 0$ (slump-type crisis). Equivalently, $D = \varrho g$ shares the sign of g while $\varrho > 0$.

For (3), suppose there exists I with $\varrho(\omega \mid \Lambda) \geq \eta > 0$ on I and $\mathcal{O}(\omega_0 \mid \Lambda) \subset I$. By continuity of MSM off $\mathcal{K}$, we can pick a common neighborhood of the saddle where $\varrho \geq \eta/2$. Then for $0 < |g| < \varepsilon$,

$$\dot{E} \;\geq\; c\,E \quad \text{for some } c > 0,$$

giving $E(t) \geq E(0)e^{ct}$ and strict divergence at each visit; no local reabsorption is possible under Λ.

For (4), along any solution ω_t,

$$\mathcal{L}_\Omega\varrho(\omega_t \mid \Lambda) \;=\; \frac{d}{dt}\varrho(\omega_t \mid \Lambda) \;=\; \frac{\partial\varrho}{\partial\theta}\frac{d\theta}{d\omega}(\omega_t \mid \Lambda)\,\Omega(\omega_t \mid \Lambda) \;+\; \frac{\partial\varrho}{\partial\omega}(\omega_t \mid \Lambda).$$

If $\mathcal{L}_\Omega \varrho \geq 0$ on $\{\varrho > 0\}$, the orbit cannot reach $\{\varrho \leq 0\}$, so crisis persists. Conversely, if under some Λ' one has $\mathcal{L}_{\Omega'} \varrho \leq -\eta' < 0$ on $\{\varrho > 0\}$, then $\varrho(\omega_t \mid \Lambda')$ falls to ≤ 0 in time $t^* \leq \varrho(\omega_0 \mid \Lambda')/\eta'$, after which Claim 4.7.1 implies $\dot{E} \leq 0$ locally and the trajectory is reabsorbed. In the benchmark $\partial\varrho/\partial\omega = 0$, the condition reduces to $\frac{\partial\varrho}{\partial\theta}\frac{d\theta}{d\omega}\Omega < 0$ on $\{\varrho > 0\}$, which uses Proposition 3.5 for $d\theta/d\omega$.

This proves the four statements and the theorem. ■

5 Conclusions

The core contribution of this chapter is the development of a novel theoretical framework that provides a Marxist reformulation of the Sraffian Supermultiplier, with micro-foundations rooted in profit-oriented behavior of capitalist firms as cornerstone. Building on a similar SSM demand-led closure, it addresses its two main weaknesses: i) neglecting Harrodian instability and ii) assuming balanced growth rate in the long-run. In the MSM, instability and unbalanced growth in the long run, stagnation tendencies, and the plausibility of crisis are embedded directly in the model. In this setting, instability becomes a normal condition of capital accumulation.

Three novelties are introduced upon this results. First, while accepting the Sraffian closure of endogenous savings, the framework also incorporates Harrodian instability, producing a bi-causal accumulation–output relation $\Delta Y \leftrightarrow \Delta K$. Second, it allows the formal integration of a non-linear dynamic system where capacity utilization and the investment share are co-determined variables. Third, this unstable co-determination unfold under a taxonomy which allows to differentiate between stagnation tendencies, crisis, regime crisis and its institutionally and technologically dependent resolution.

The main break with Neo-Kaleckian growth models lies in their reliance on conventions, hysteresis, or micro-foundations such as the choice of shift or technique to guarantee long-run convergence to stability. In contrast, here a dynamic equation of capacity utilization is specified as a law of motion, resulting in stagnation tendencies and intrinsic instability. Utilization at normal levels becomes a methodological assumption, but analytically exogenous demand sets fixed points that deviate from it, thereby breaking Say's Law in the long run in contrast of the standard portrait of marxist political economy by post-keynesian literature Blecker and Setterfield (2019). Moreover, the MSM framework develops this argument without neglecting neither recurring to time lags as in the capitatl circuit literature Foley (1982); Basu (2014), neither to labour value theory. This have been done with the purpose to engage heterodox economic more broadly with the argument but also to proceed in future research levearing in the developments of the Srafian Super Multiplier literature.

The notion of *"autonomous demand"* of SSM is here rejected. Instead, demand is classified into induced and capacity-generating components, while non-induced, non-capacity-generating expenditures are treated not as truly autonomous but *as an analytical device*. In this case, government expenditures and export demand are considered in such a way that the framework can identify plausible counter-cyclical policies against trade cycles. Exogenous demand is therefore not the driver of long-run growth but defines fixed points of long-run equilibrium, which may be stable or unstable. Growth can be driven by such demand, but only conditionally and depending on the policy mix; it is not built into the model by construction. Rather, it becomes a political problem at the shadow of the world market.

This yields several contributions to Marxist political economy. Without relying on the two-department framework, the emphasis on unbalanced growth through the choice of technique highlights two of Marx's central concerns: disproportionality and realization. In the MSM, these concerns become orthogonal. Under specific institutional and technological conditions, crises arise within unstable structures marked by stagnation tendencies, as disproportional absorptive capacities hinder the realization of accumulation demand.

The analytical results permit a taxonomy of outcomes: (i) stagnation tendencies marked by persistent under- or over-accumulation, (ii) partial crises, where instability is realized but contained within the same structural features, and (iii) structural crises, where realized instability cannot be resolved without a regime change. This taxonomy is made possible by redefining the Sraffian Supermultiplier within a Harrodian framework of instability and unbalanced growth, yielding a dynamic multiplier that allows a wide range of results, including negative ones. These include divergence of capacity utilization and realization crises, understood as widening deviations from a saddle-type long-run equilibrium. Furthermore, instability can flip depending on the investment share trajectory and the choice of technique. Harrodian reversal dynamics—rising capacity utilization coupled with a shrinking investment share—are endogenous to the accumulation process. As a result, crises due to overheating and recoveries from downturns are both plausible within this framework.

That said, the theoretical framework has limitations. First, it is analytical in scope only; distributional dynamics are considered abstractly and mapped through long-run behavioral assumptions about conflict. Second, the model's non-linear nature makes it complex to analyze. Third, it still relies on a component of demand that is analytically exogenous. However, unlike in the SSM, this is not meant to provide a *"truly autonomous"* growth driver. Instead, the exogeneity is openly acknowledged and chosen to identify methodologically plausible uses of counter-cyclical policy, especially government spending, as a stabilizing mechanism. Future research should test different policy scenarios through simulations, including the case where fiscal discipline endogenously constrains expenditures. Finally, the law of capacity building is generic and may not capture the constraints faced by peripheral economies with binding balance-of-payments conditions. Nevertheless, the structural resemblance to the SSM ensures that this avenue of research can continue, bridging Sraffian classical reconstruction with Marx's critique in a contemporary framework.

The MSM thus establishes instability and crisis not as anomalies but as intrinsic outcomes of accumulation, providing a dynamic and demand-led reformulation of long-period growth theory. By embedding Harrodian instability within a Super Multiplier analytical structure, the framework opens new ground for analyzing stagnation tendencies, reversals, and structural crises without relying on equilibrium convergence. This foundation sets the stage for the empirical work of the next chapter, where alternative measures of capacity utilization and demand composition will be constructed to evaluate the dynamics outlined here in concrete historical settings.

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A Appendix A: Comparative statics of $\theta(\omega \mid \Lambda)$

A.1 Capitalist Profit-Oriented Behavior

The maximization problem for the incremental profit rate $\hat{r}$ with respect to induced mechanization $\hat{m}$, subject to the mechanization–productivity constraint:

$$\max_{\hat{m}} \quad \hat{r} = \hat{a} - (1-\omega)\hat{m} \quad \text{s.t.} \quad \hat{a} = g(\hat{m} \mid \Lambda).$$

Hence, the corresponding Lagrangian is

$$\mathcal{L}(\hat{m}, \hat{a}, \lambda) = \hat{a} - (1-\omega)\hat{m} + \lambda\big(\hat{a} - g(\hat{m} \mid \Lambda)\big).$$

First-order conditions are:

$$\begin{aligned} \frac{\partial \mathcal{L}}{\partial \hat{a}} &= 1 + \lambda = 0 & \Rightarrow \quad \lambda = -1, \\ \frac{\partial \mathcal{L}}{\partial \hat{m}} &= -(1-\omega) - \lambda\, g_{\hat{m}}(\hat{m}, \omega \mid \Lambda) = 0. \end{aligned}$$

Substituting $\lambda = -1$ yields the optimality condition:

$$g_{\hat{m}}(\hat{m}^*(\omega \mid \Lambda), \omega \mid \Lambda) = 1 - \omega \tag{A.1}$$

Given monotonicity of g ($g_{\hat{m}} > 0$), the optimal mechanization rate can be expressed as

$$\hat{m}^*(\omega \mid \Lambda) = g'^{-1}(1 - \omega \mid \Lambda), \tag{A.2}$$

which in turn implies

$$\hat{a}^*(\omega \mid \Lambda) = g(\hat{m}^*(\omega \mid \Lambda) \mid \Lambda), \qquad \theta(\omega \mid \Lambda) = \frac{g(\hat{m}^*(\omega \mid \Lambda) \mid \Lambda)}{\hat{m}^*(\omega \mid \Lambda)}. \tag{A.3}$$

Thus $\theta(\omega \mid \Lambda)$ is endogenously determined by distributive conflict ω and the prevailing regime

Λ.

All partial derivatives of g are evaluated at $\big(\hat{m}^*(\omega \mid \Lambda), \omega \mid \Lambda\big)$ unless otherwise indicated. We write $g_{\hat{m}} = \partial g/\partial \hat{m}$, $g_\omega = \partial g/\partial \omega$, $g_{\hat{m}\hat{m}} = \partial^2 g/\partial \hat{m}^2$, $g_{\hat{m}\omega} = \partial^2 g/(\partial \hat{m}\partial \omega)$, etc.

A.2 Capitalist Optimization Problem

Step 1: Total derivative of θ

$$\theta(\omega \mid \Lambda) = \frac{g(\hat{m}^*(\omega \mid \Lambda), \omega \mid \Lambda)}{\hat{m}^*(\omega \mid \Lambda)} \tag{A.4}$$

$$\frac{d\theta}{d\omega} = \frac{1}{\hat{m}^*}\Big(g_{\hat{m}}\frac{d\hat{m}^*}{d\omega} + g_\omega\Big) - \frac{g}{(\hat{m}^*)^2}\frac{d\hat{m}^*}{d\omega} \tag{A.5}$$

Using (**??**) and (**??**) (so that $g_{\hat{m}} = 1 - \omega$ and $g = \theta\,\hat{m}^*$) we get:

$$\frac{d\theta}{d\omega} = \frac{(1-\omega)\,\dfrac{d\hat{m}^*}{d\omega} + g_\omega}{\hat{m}^*} - \frac{\theta\,\hat{m}^*}{(\hat{m}^*)^2}\,\frac{d\hat{m}^*}{d\omega} \tag{A.6}$$

$$= \frac{\big[(1-\omega) - \theta(\omega \mid \Lambda)\big]\,\dfrac{d\hat{m}^*}{d\omega} + g_\omega}{\hat{m}^*}. \tag{A.7}$$

Step 2: Implicit differentiation of the FOC

Differentiate (A.1) w.r.t. ω:

$$\frac{d}{d\omega}\,g_{\hat{m}}(\hat{m}^*, \omega) = g_{\hat{m}\hat{m}}\,\frac{d\hat{m}^*}{d\omega} + g_{\hat{m}\omega} = -1.$$

Hence

$$g_{\hat{m}\hat{m}}\,\frac{d\hat{m}^*}{d\omega} = -1 - g_{\hat{m}\omega} \quad \Rightarrow \quad \boxed{\frac{d\hat{m}^*}{d\omega} = -\,\frac{1 + g_{\hat{m}\omega}}{g_{\hat{m}\hat{m}}}}. \tag{A.8}$$

(For an interior maximum the SOC requires $g_{\hat{m}\hat{m}} < 0$.)

Step 3: $\dfrac{d\theta}{d\omega}$ General form

Substitute (A.8) into (A.7) to obtain the general expression:

$$
\begin{aligned}
\frac{d\theta(\omega \mid \Lambda)}{d\omega} &= \frac{\big[(1-\omega) - \theta(\omega \mid \Lambda)\big]\left(-\dfrac{1+g_{\hat{m}\omega}}{g_{\hat{m}\hat{m}}}\right) + g_\omega}{\hat{m}^*} \\
&= -\,\frac{\big[(1-\omega) - \theta(\omega \mid \Lambda)\big]\,(1 + g_{\hat{m}\omega}(\hat{m}^*, \omega \mid \Lambda))}{g_{\hat{m}\hat{m}}(\hat{m}^*, \omega \mid \Lambda)\,\hat{m}^*(\omega \mid \Lambda)} \;+\; \frac{g_\omega(\hat{m}^*, \omega \mid \Lambda)}{\hat{m}^*(\omega \mid \Lambda)}. \qquad \text{(A.9)}
\end{aligned}
$$

Step 4: Benchmark: Mechanization specialization without direct or cross dependence on distribution ω)

If g does not depend directly on ω at the given Λ, set

$$g_\omega = 0, \qquad g_{\hat{m}\omega} = 0.$$

Then (A.8) reduces to $\dfrac{d\hat{m}^*}{d\omega} = -1/g_{\hat{m}\hat{m}}$ and (A.7) yields the benchmark formula

$$\boxed{\frac{d\theta(\omega \mid \Lambda)}{d\omega} = -\,\frac{(1-\omega) - \theta(\omega \mid \Lambda)}{g_{\hat{m}\hat{m}}(\hat{m}^* \mid \Lambda)\,\hat{m}^*(\omega \mid \Lambda)}} \qquad \text{(A.10)}$$

with the sign of the denominator determined by the SOC $g_{\hat{m}\hat{m}}(\hat{m}^* \mid \Lambda) < 0$.

Equation (A.9) identifies two distributional channels on the choice of technique: (i) an implicit channel through $\dfrac{d\hat{m}^*}{d\omega} = \dfrac{1+g_{\hat{m}\omega}}{g_{\hat{m}\hat{m}}}$ (first term) and (ii) a direct distributional effect $g_\omega/\hat{m}^*$ (second term). The benchmark specification used in this model, is setting the direct channel to zero and focus on the implicit channel driven by solely by technology expressed in the curvature $g_{\hat{m}\hat{m}}$ of the mechanization function, and the average–marginal gap $(1-\omega)-\theta$ of mechanization gains. For an interior maximum the SOC $g_{\hat{m}\hat{m}} < 0$ is required, but not necessarily hold locally given that $g_{\hat{m}\hat{m}} > 0$ is not precluded locally in equation (A.10).

B Analysis of the Dynamic System of Capacity Utilization u and the Investment Share ϕ

This appendix proceeds in four steps to analyze the non-linear dynamic system of capacity utilization and investment share: (1) identify the fixed points of the dynamic system; (2) demonstrate the economic meaningfulness of these fixed points; (3) derive the trace, determinant, and discriminant of the Jacobian; (3) develop analysis for cases around multiple equilibrium fixed points.

B.1 Step 1: Existence of Two Interior Fixed Points

Recall the dynamic system defined by equations (4.1) and (4.2), we get at the (ϕ, u) plane:

1. The $\dot{\phi} = 0$ nullcline is given by:

$$0 = \big[\hat{z} - \theta\gamma\,(\mu - 1)\big]\,(\gamma\mu - 1) + (1-\theta)\gamma(\mu - 1)$$

Developing step by step:

$$\begin{aligned}
0 &= \hat{z}\,(\gamma\mu) \;-\; \hat{z} \;-\; \theta\gamma\,(\mu-1)\,(\gamma\mu) \;+\; \theta\gamma\,(\mu-1) \;+\; (1-\theta)\gamma\,(\mu-1) \\
&= \gamma\,\hat{z}\,\mu \;-\; \hat{z} \;-\; \theta\gamma^2\big(\mu^2-\mu\big) \;+\; \theta\gamma\,(\mu-1) \;+\; (1-\theta)\gamma\,(\mu-1) \\
&= \gamma\,\hat{z}\,\mu \;-\; \hat{z} \;-\; \theta\gamma^2\,\mu^2 + \theta\gamma^2\,\mu \;+\; \big[\theta + (1-\theta)\big]\gamma\,(\mu-1) \\
&= \gamma\,\hat{z}\,\mu \;-\; \hat{z} \;-\; \theta\gamma^2\,\mu^2 + \theta\gamma^2\,\mu \;+\; \gamma\,(\mu-1) \\
&= -\,\theta\,\gamma^2\,\mu^2 \;+\; \big[\gamma\,\hat{z} + \theta\gamma^2 + \gamma\big]\,\mu \;-\; (\hat{z}+\gamma) \\
&= -\,\theta\,\gamma^2\,\mu^2 \;+\; \gamma\big[\hat{z} + \gamma\theta + 1\big]\,\mu \;-\; (\hat{z}+\gamma).
\end{aligned}$$

Thus the $\dot{\phi} = 0$ nullcline can be written as the quadratic equation.

$$\boxed{\dot{\phi}_{\dot{\phi}=0}(\mu) = -\theta\gamma^2\mu^2 + \gamma\big[\hat{z} + \gamma\theta + 1\big]\mu - (\hat{z} + \gamma)} \tag{B.1}$$

2. The $\dot{\mu} = 0$ nullcline with $\mu > 0$ is given by:

$$\hat{\mu} = \hat{z} + \frac{\phi}{s - \phi + m}\hat{\phi} - \theta\gamma(\mu - 1)$$

$$0 = \hat{z} + \frac{\phi}{s - \phi + m}\hat{\phi} - \theta\gamma(\mu - 1)$$

$$\frac{\phi}{s - \phi + m}\hat{\phi} = \theta\gamma(\mu - 1) - \hat{z}$$

$$\dot{\phi} = (s - \phi + m)[\theta\gamma(\mu - 1) - \hat{z}]$$

Such a dynamic equation of $\dot{\phi}$ emerges at the nullcline $\dot{\mu}$,i.e., $\dot{\phi}_{\dot{\mu}=0}(\mu, \phi)$ which can be expressed as:

$$\boxed{\dot{\phi}_{\dot{\mu}=0}(\mu, \phi) = (s - \phi + m)[\theta\gamma(\mu - 1) - \hat{z}]} \tag{B.2}$$

This function represents the (ϕ, μ) space which values of $\dot{\phi}$ sustain capacity utilization constant. To find the intersection of $\dot{\phi}_{\dot{\mu}=0}(\mu, \phi) = 0$ with the nullcline $\dot{\phi}_{\dot{\phi}=0}(\mu)$ we need to recall that the condition for the Super Multiplier to be positive and finite is $0 < \phi < m + s$, so the left hand term of the function is always positive, which implies that:

$$0 = \theta\gamma(\mu - 1) - \hat{z}$$

$$\mu - 1 = \frac{\hat{z}}{\theta\gamma}$$

$$\mu = 1 + \frac{\hat{z}}{\theta\gamma}$$

Thus, the $\dot{\mu} = 0$ nullcline corresponds to two vertical lines in the (ϕ, μ) space. Recall that $\hat{z} > 0$ or $\hat{z} < 0$ Hence,

$$\boxed{\mu^* = \begin{cases} 1 + \dfrac{|\hat{z}|}{\theta\,\gamma} & \text{if} \quad \hat{z} > 0, \\ 1 - \dfrac{|\hat{z}|}{\theta\,\gamma} & \text{if} \quad \hat{z} < 0 \end{cases}} \tag{B.3}$$

B.2 Step 2: Conditions for Meaningful Economic Interpretations of the Fixed Points Solutions

The steady states of the system are located at the intersection of this vertical line with the quadratic curve defined by the $\dot{\phi}(\mu) = 0$ nullcline. So, using the quadratic equation formula we can infer that:

$$\dot{\phi}(\mu) = -\,\theta\,\gamma^2\,\mu^2 + \gamma\big[\hat{z} + \gamma\theta + 1\big]\,\mu - (\hat{z} + \gamma)$$

So, we can express as a quadratic equation:

$$0 = a\mu^2 + b\mu + c \tag{B.4}$$

Where,

$$a = -\theta\,\gamma^2 \quad b = \gamma\big[\hat{z} + \gamma\theta + 1\big] \quad c = -(\hat{z} + \gamma)$$

Hence, the solutions can be expressed as:

$$\mu_{\pm} = \frac{\hat{z} + \gamma\theta + 1 \pm \sqrt{(\hat{z} + \gamma\theta + 1)^2 - 4\theta\gamma^2(\hat{z} + \gamma)}}{2\theta\,\gamma} \tag{B.5}$$

Where,

$$\mu_{\pm} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

First, we analyze the discriminant expression: $\Delta = b^2 - 4ac$, which interestingly, can be expressed as a quadratic function of $\hat{z}$.

$$\Delta(\hat{z}) = (\hat{z} + \gamma\theta + 1)^2 - 4\,\theta\gamma^2(\hat{z} + \gamma) \geq 0 \tag{B.6}$$

To re-write $\hat{z}$ as a quadratic equation, first take the left hand side of the equation (B.6) and develop algebraically as follows:

$$(\hat{z} + \gamma\theta + 1)^2 = \hat{z}^2 + 2(\gamma\theta + 1)\,\hat{z} + (\gamma\theta + 1)^2$$

Such that the $\Delta(\hat{z})$ becomes:

$$\Delta(\hat{z}) = \hat{z}^2 + \left[2(\gamma\theta + 1) - 4\,\theta\,\gamma^2\right]\hat{z} + \left[(\gamma\theta + 1)^2 - 4\,\theta\,\gamma^3\right]$$

Once again we can identity a quadratic equation expression, in this case for $\Delta(\hat{z})$, the discriminant equation corresponding to (**??**).

$$\Delta(\hat{z}) = a_\Delta \hat{z}^2 + b_\Delta \hat{z} + c_\Delta \tag{B.7}$$

Where,

$$\begin{aligned} a_\Delta &= 1 \\ b_\Delta &= 2(\gamma\,\theta + 1) - 4\,\theta\,\gamma^2 \\ c_\Delta &= (\gamma\,\theta + 1)^2 - 4\,\theta\,\gamma^3 \end{aligned}$$

Hence, considering $a_\Delta = 1$, defining the discriminant of equation (**??**) as $D_\Delta = b_\Delta^2 - 4a_\Delta c_\Delta$ we can characterize the set $\{\hat{z} : \Delta(\hat{z}) \geq 0\}$ as follows:

If $D_\Delta < 0$, $\Delta(\hat{z})$ does not have real solutions in the $(\hat{z}, \Delta(\hat{z}))$ plane. This entails that the quadratic U-shaped function never cross the $\hat{z}$ axis, hence $\Delta(\hat{z})$ necessarily should be placed above the $\hat{z} = 0$ axis. As being U-shaped placed below it would necessarily cross the axis having real roots. Hence,

$$\Delta(\hat{z}) > 0 \quad \forall\, \hat{z} \in \mathbb{R}, \tag{B.8}$$

If $D_\Delta \geq 0$, let $r_1 < r_2$ be the two real roots of $\Delta(\hat{z}) = 0$. Since $a_\Delta = 1 > 0$, the parabola is positive outside the interval $[r_1, r_2]$. Thus,

$$\{\hat{z} : \Delta(\hat{z}) \geq 0\} \;=\; (-\infty, r_1] \;\cup\; [r_2, \infty) \tag{B.9}$$

Now we express the threshold for the real roots of the function $\Delta(\hat{z}) = 0$ using the formula of the quadratic equation:

$$r_{1,2} = \frac{-b_\Delta \pm \sqrt{b_\Delta^2 - 4a_\Delta c_\Delta}}{2a}$$

Squaring b_Δ and developing the expression:

$$\begin{aligned} b_\Delta^2 &= \big[2(\gamma\;\theta + 1)\big]^2 - 2 \cdot 2(\gamma\theta + 1)4\theta\gamma^2 + (4\theta\gamma^2)^2 \\ &= 4(\gamma\;\theta + 1)^2 - 16(\gamma\;\theta + 1)\theta\gamma^2 + 16\theta^2\gamma^4 \end{aligned}$$

While $4a_\Delta c_\Delta$:

$$4a_\Delta c_\Delta = 4[(\gamma\,\theta + 1)^2 - 4\,\theta\,\gamma^3]$$

Therefore, the discriminant of the quadratic function of $\Delta(\hat{z})$ becomes:

$$
\begin{aligned}
b_\Delta^2 - 4a_\Delta c_\Delta &= 4(\gamma\ \theta+1)^2 - 16(\gamma\ \theta+1)\theta\gamma^2 + 16\theta^2\gamma^4 - 4[(\gamma\,\theta+1)^2 - 4\,\theta\,\gamma^3] \\
&= \cancel{4(\gamma\ \theta+1)^2 - 4(\gamma\,\theta+1)^2} - 16(\gamma\ \theta+1)\theta\gamma^2 + 16\theta^2\gamma^4 + 16\,\theta\,\gamma^3 \\
&= -16(\gamma\ \theta+1)\theta\gamma^2 + 16\theta^2\gamma^4 + 16\,\theta\,\gamma^3 \\
&= 16\theta\gamma^2[-(\gamma\theta+1) + \theta\gamma^2 + \gamma] \\
&= 16\theta\gamma^2(\theta\gamma^2 - \gamma\theta + \gamma - 1) \\
&= 16\theta\gamma^2[\theta\gamma(\gamma-1) - (\gamma+1)] \\
&= 16\theta\gamma^2[(\gamma-1)(\theta\gamma+1)]
\end{aligned}
$$

Hence, the condition for $\Delta(\hat{z}) \geq 0$ is:

$$
\boxed{\Delta(\hat{z}) > \begin{cases} \text{if } \gamma < 1 \implies D_\Delta < 0 \quad \forall\,\hat{z} \\ \text{if } \gamma > 1 \implies D_\Delta > 0 \quad \text{ for } \hat{z} \leq r_1 \quad \text{or} \quad \hat{z} \geq r_2 \end{cases}} \tag{B.10}
$$

B.3 Step 3: Derivation of the General form of the Jacobian, Trace, and Discriminant for Stability Analysis around Fixed Points

Define the Jacobian matrix of the dynamic system as:

$$
J(u,\phi) = \begin{pmatrix} \dfrac{\partial\dot{\mu}}{\partial\mu} & \dfrac{\partial\dot{\mu}}{\partial\phi} \\ \dfrac{\partial\dot{\phi}}{\partial\mu} & \dfrac{\partial\dot{\phi}}{\partial\phi} \end{pmatrix}.
$$

and we denote:

$$
J(\mu,\phi) \;=\; \begin{pmatrix} F_\mu(\mu,\phi) & F_\phi(\mu,\phi) \\ G_\mu(\mu,\phi) & G_\phi(\mu,\phi) \end{pmatrix}
$$

Where,

$$F(\mu,\phi) \equiv \dot{\mu} = \mu\big[\hat{z} + \tfrac{1}{s-\phi+m}\,\dot{\phi} - \theta\,\gamma\,(\mu-1)\big]$$
$$G(\mu,\phi) \equiv \dot{\phi} = \phi\big[(\gamma\,\mu-1)\,\dot{\mu} + (1-\theta)\,\gamma\,(\mu-1)\big]$$

$$F(\mu,\phi) = \mu\left[\hat{z} + \frac{1}{s-\phi+m}\dot{\phi} - \theta\,\gamma\,(\mu-1)\right]$$

At steady state, $\dot{\phi} = 0$:

$$\begin{aligned}
F(\mu,\phi) &= \mu\,[\hat{z} - \theta\,\gamma\,(\mu-1)] \\
F_\mu &= \frac{\partial}{\partial\mu}\,[\mu\,(\hat{z} - \theta\,\gamma\,(\mu-1))] \\
&= (\hat{z} - \theta\,\gamma\,(\mu-1)) + \mu\,(-\theta\,\gamma) \\
&= \hat{z} - \theta\,\gamma\,(\mu-1+\mu) \\
&= \hat{z} - \theta\,\gamma\,(2\mu-1)
\end{aligned}$$

$$\begin{aligned}
F(\mu,\phi) &= \mu\,[\hat{z} - \theta\,\gamma\,(\mu-1)] \\
F_\phi &= \frac{\partial}{\partial\phi}\,[\mu\,(\hat{z} - \theta\,\gamma\,(\mu-1))] \\
&= 0
\end{aligned}$$

$$G(\mu,\phi) = \phi\big[(\gamma\mu-1)\dot{\mu} + (1-\theta)\gamma(\mu-1)\big]$$

At steady state, $\dot{\mu} = 0$

$$
\begin{aligned}
G(\mu,\phi) &= \phi(1-\theta)\gamma(\mu-1)\\
G_\mu &= \frac{\partial}{\partial\mu}\left[\phi(1-\theta)\gamma(\mu-1)\right]\\
&= \phi(1-\theta)\gamma
\end{aligned}
$$

$$
\begin{aligned}
G_\phi &= \frac{\partial}{\partial\phi}\left[\phi(1-\theta)\gamma(\mu-1)\right]\\
&= (1-\theta)\gamma(\mu-1)
\end{aligned}
$$

Therefore, the jacobian matrix becomes defined by:

$$
J(\mu,\phi) \;=\; \begin{pmatrix} \hat{z}-\theta\,\gamma\,(2\mu-1) & 0 \\ \phi(1-\theta)\gamma & (1-\theta)\gamma(\mu-1) \end{pmatrix} \tag{B.11}
$$

I abstract to take the steady state values of for μ^* and ϕ^* because as the former is featured by a multiple equilibrium below or above normal capacity utilization, it would come easier to analyze accumulation regimes in terms of cases for θ first, and over/under capacity utilization in a second moment.

In this line, we can define a general equation for the trace of the Jacobian as follows:

$$
\begin{aligned}
\operatorname{tr} J(\mu,\phi) &= F_\mu + G_\phi\\
&= \left[\hat{z}-\theta\,\gamma\,(2\mu-1)\right] + (1-\theta)\,\gamma\,(\mu-1)\\
&= \hat{z} - 2\theta\gamma\mu + \theta\gamma + (1-\theta)\gamma\mu - (1-\theta)\gamma\\
&= \hat{z} + \gamma(2\theta-1) + \gamma(1-3\theta)\,\mu
\end{aligned}
$$

Such that we state the general form of the trace as follows:

$$\boxed{\operatorname{tr} J(\mu,\phi) = \hat{z} + \gamma(2\theta - 1) + \gamma(1 - 3\theta)\,\mu} \tag{B.12}$$

On the other hand, as the first row second column of the jacobian matrix is zero, the general form of the jacobian matrix equation becomes the product of the diagonal, which again is a function of the same terms.

$$\begin{aligned}
\det J(\mu,\phi) &= F_\mu G_\phi \\
&= [\hat{z} - \theta\,\gamma\,(2\mu - 1)]\;(1-\theta)\,\gamma\,(\mu - 1) \\
&= [(\hat{z} + \theta\gamma) - 2\theta\gamma\,\mu]\;(1-\theta)\,\gamma\,(\mu - 1) \\
&= (1-\theta)\,\gamma\,[(\hat{z} + \theta\gamma)(\mu - 1) - 2\theta\gamma\,\mu(\mu - 1)] \\
&= (1-\theta)\gamma\left[\hat{z}\mu - \hat{z} + \theta\gamma\mu - \theta\gamma - 2\theta\gamma\mu^2 + 2\theta\gamma\mu\right] \\
&= (1-\theta)\gamma\left[-2\theta\gamma\mu^2 + (\hat{z} + \theta\gamma + 2\theta\gamma)\mu - (\hat{z} + \theta\gamma)\right]
\end{aligned}$$

$$\det J(\mu,\phi) = (1-\theta)\,\gamma\;\Big[\;\underbrace{-2\,\theta\gamma\,\mu^2}_{\text{quadratic in }\mu} \;+\; \underbrace{(\hat{z} + 3\,\theta\gamma)\,\mu}_{\substack{\text{linear in }\mu,\\ \text{contains }\hat{z}}} \;-\; \underbrace{(\hat{z} + \theta\gamma)}_{\text{constant + linear in }\hat{z}}\;\Big] \tag{B.13}$$

Finally, we identify the general form of the discriminant of the Jacobian J, Δ_J. From squaring equation (B.12), we get:

$$\operatorname{tr} J(\mu,\phi)^2 = [\hat{z} + \gamma(2\theta - 1) + \gamma(1 - 3\theta)\mu]^2$$

$$\text{Define:}\quad X = \hat{z} + \gamma(2\theta - 1), \qquad Y = \gamma(3\theta - 1)$$

$$\begin{aligned}
\Delta(\mu,\phi) &= \left[\hat{z} + \gamma(2\theta - 1) + \gamma(1 - 3\theta)\,\mu\right]^2 \;-\; 4\,\det J(\mu,\phi) \\
(\operatorname{tr} J)^2 &= [X - Y\mu]^2 \\
&= [X - \gamma(1+\theta)\,\mu]^2 \\
&= [X - Y\mu]^2 \\
&= X^2 - 2XY\mu + Y^2\mu^2
\end{aligned}$$

$$\text{Define:}\quad Z = \hat{z} + \theta\gamma, \quad W = \gamma\theta, \quad K = (1-\theta)\gamma$$

$$\begin{aligned}
-4\,\det J(\mu,\phi) &= -4(1-\theta)\gamma\big[(\hat{z} + \theta\gamma)(\mu - 1) - 2\theta\gamma\mu(\mu - 1)\big] \\
&= -4K\big[Z(\mu - 1) - 2W\mu(\mu - 1)\big] \\
&= -4K\big[Z\mu - Z - 2W\mu^2 + 2W\mu\big] \\
&= -4K(-Z) \;-\; 4KZ\mu \;+\; 8KW\mu^2 \;-\; 8KW\mu \\
&= 4KZ - 4K(Z + 2W)\mu \;+\; 8KW\mu^2 \\
A &= Y^2 + 8KW \\
Y^2 + 8KW &= \gamma^2(3\theta - 1)^2 + 8\gamma^2\theta(1-\theta) \\
&= \gamma^2\big[(3\theta - 1)^2 + 8\theta - 8\theta^2\big] \\
&= \gamma^2\big[9\theta^2 - 6\theta + 1 + 8\theta - 8\theta^2\big] \\
&= \gamma^2\big[\theta^2 + 2\theta + 1\big] \\
A &= \boxed{\gamma^2(1+\theta)^2} \\
B &= -\big[2XY + 4K(Z + 2W)\big] \\
2XY &= 2[\hat{z} + \gamma(2\theta - 1)]\,\big[\gamma(3\theta - 1)\big] \\
&= 2\gamma(3\theta - 1)\,\hat{z} \;+\; 2\gamma^2(2\theta - 1)(3\theta - 1) \\
&= 2\gamma(3\theta - 1)\,\hat{z} \;+\; \gamma^2(12\theta^2 - 10\theta + 2) \\
Z + 2W &= \hat{z} + 3\gamma\theta \\
4K(Z + 2W) &= 4(1-\theta)\gamma(\hat{z} + 3\gamma\theta) = 4\gamma(1-\theta)\,\hat{z} \;+\; 12\gamma^2\theta(1-\theta)
\end{aligned}$$

$$
\begin{aligned}
\Rightarrow \quad 2XY + 4K(Z+2W) &= -\Big([2\gamma(3\theta-1) + 4\gamma(1-\theta)]\hat{z} \ + \ \gamma^2[12\theta^2 - 10\theta + 2 + 12\theta - 12\theta^2] \Big) \\
&= -\Big(2\gamma[(3\theta-1) + 2(1-\theta)]\hat{z} \ + \ 2\gamma^2(\theta+1) \Big) \\
&= -\Big(2\gamma(\theta+1)\hat{z} \ + \ 2\gamma^2(\theta+1) \Big) \\
B &= \boxed{-2\gamma(1+\theta)\,(\hat{z}+\gamma)} \\
C &= X^2 + 4KZ \\
X^2 + 4KZ &= \big[\hat{z} + \gamma(2\theta-1)\big]^2 \ + \ 4\gamma(1-\theta)\big[\hat{z} + \theta\gamma\big] \\
&= \hat{z}^2 + 2\gamma(2\theta-1)\hat{z} + \gamma^2(2\theta-1)^2 \ + \ 4\gamma(1-\theta)\hat{z} + 4\gamma^2\theta(1-\theta) \\
&= \hat{z}^2 + 2\gamma\big[(2\theta-1) + 2(1-\theta)\big]\hat{z} \ + \ \gamma^2\big[(2\theta-1)^2 + 4\theta(1-\theta)\big] \\
&= \hat{z}^2 + 2\gamma\,\hat{z} + \gamma^2 \\
C &= \boxed{(\hat{z}+\gamma)^2}
\end{aligned}
$$

Hence, we can define the discriminant $\Delta_{J(\phi,\mu)} = (\text{tr } J)^2 - 4 \det J(\mu,\phi)$ as a quadratic function in the $(\Delta_{J(\phi,\mu)}, \mu)$ space.

$$
\Delta_{J(\phi,\mu)}(\mu) = A\mu^2 - B\mu + C \tag{B.14}
$$

Which yields,

$$
\begin{aligned}
A\mu^2 - B\mu + C &= \gamma^2(1+\theta)^2\mu^2 - \big[-2\gamma(1+\theta)(\hat{z}+\gamma)\big]\mu + (\hat{z}+\gamma)^2 \\
&= \gamma^2(1+\theta)^2\mu^2 + 2\gamma(1+\theta)(\hat{z}+\gamma)\mu + (\hat{z}+\gamma)^2
\end{aligned}
$$

Such that the general form of the discriminant expressed in terms of parameters takes the following functional form:

$$
\Delta_{J(\phi,\mu)}(\mu) = \big[\gamma(1+\theta)\mu + (\hat{z}+\gamma)\big]^2 \tag{B.15}
$$

So the Jacobian discriminant is a perfect square, hence non-negative for all u, i.e. $\Delta_{J(\phi,\mu)}(\mu) \geq 0$ always holds.

B.4 Step 4: Stability Analysis around Fixed Points

$\det J(\mu_+, \phi)$ Analysis

Let $\mu_+^* = 1 + |\delta|$, $\mu_+^* - 1 = |\delta|$, and $\hat{z} = \gamma\theta\,|\delta|$, plug into the trace general form equation, and consider $\kappa(\theta) = \dfrac{\theta}{1-2\theta}$ plug into the general form of the determinant of the jacobian (B.13) valued at μ_+^*:

$$
\begin{aligned}
\det J(\mu_+, \phi) &= (1-\theta)\,\gamma\Big[-2\,\theta\gamma\,(1+|\delta|)^2 + (\gamma\theta\,|\delta| + 3\,\theta\gamma)\,(1+|\delta|) - (\gamma\theta\,|\delta| + \theta\gamma)\Big] \\
&= (1-\theta)\,\gamma\Big[-2\theta\gamma - 4\theta\gamma|\delta| - 2\theta\gamma|\delta|^2 + 3\theta\gamma + 4\theta\gamma|\delta| + \theta\gamma|\delta|^2 - \theta\gamma|\delta| - \theta\gamma\Big] \\
&= (1-\theta)\,\gamma\Big[-\theta\gamma|\delta| - \theta\gamma|\delta|^2\Big] \\
&= (1-\theta)\,\gamma\Big[-\theta\gamma|\delta|\,(1+|\delta|)\Big] \\
&= \theta(\theta-1)\,\gamma^2\,|\delta|\,(1+|\delta|)
\end{aligned}
$$

$$
\boxed{\det J(\mu_+, \phi) = \theta(\theta-1)\,\gamma^2\,|\delta|\,(1+|\delta|)} \tag{B.16}
$$

$\det J(\mu_-, \phi)$ Analysis

Let $\mu_-^* = 1 - |\delta|$, $\mu_-^* - 1 = -|\delta|$, and $\hat{z} = -\gamma\theta\,|\delta|$, plug into the trace general form equation, and consider $\kappa(\theta) = \dfrac{\theta}{1-2\theta}$ we get:

$$
\begin{aligned}
\det J(\mu_-, \phi) &= (1-\theta)\,\gamma\Big[-2\,\theta\gamma\,(1-|\delta|)^2 + \big(-\gamma\theta|\delta| + 3\,\theta\gamma\big)\,(1-|\delta|) - \big(-\gamma\theta|\delta| + \theta\gamma\big)\Big] \\
&= (1-\theta)\,\gamma\Big[-2\theta\gamma + 4\theta\gamma|\delta| - 2\theta\gamma|\delta|^2 + 3\theta\gamma - 4\theta\gamma|\delta| + \theta\gamma|\delta|^2 - \theta\gamma + \theta\gamma|\delta|\Big] \\
&= (1-\theta)\,\gamma\Big[\,\theta\gamma|\delta| - \theta\gamma|\delta|^2\,\Big] \\
&= \theta(1-\theta)\,\gamma^2\,|\delta|\,(1-|\delta|)
\end{aligned}
$$

$$
\boxed{\det J(\mu_-, \phi) = \theta(1-\theta)\,\gamma^2\,|\delta|\,(1-|\delta|)} \tag{B.17}
$$

$\operatorname{tr} J(\mu_+, \phi)$ **Analysis**

Now recall the expression for the trace from equation (B.12):

$$
\operatorname{tr} J(\mu, \phi) = \hat{z} + \gamma(2\theta - 1) + \gamma(1-3\theta)\,\mu \tag{B.18}
$$

Let $\mu_+ = 1 + |\delta|$, $\mu_+ - 1 = |\delta|$, and $\hat{z} = \gamma\theta\,|\delta|$ and define $\kappa(\theta) = \dfrac{\theta}{1-2\theta}$, plug into the determinant and re-arrange so we get:

$$
\begin{aligned}
\operatorname{tr} J(\mu_+, \phi) &= \hat{z} + \gamma(2\theta - 1) + \gamma(1-3\theta)\,\mu_+ \\
&= \gamma\theta|\delta| + \gamma(2\theta-1) + \gamma(1-3\theta)\,(1+|\delta|) \\
&= \gamma\big[(1-2\theta)\,|\delta| - \theta\big] \\
&= \gamma(1-2\theta)\left(|\delta| - \frac{\theta}{1-2\theta}\right) \\
&= \gamma(1-2\theta)\,\big(|\delta| - \kappa(\theta)\big).
\end{aligned}
$$

$$
\boxed{\operatorname{tr} J(\mu_+, \phi) \;=\; \gamma(1-2\theta)\,\big(|\delta| - \kappa(\theta)\big)} \tag{B.19}
$$

$\operatorname{tr} J(\mu_-, \phi)$ **Analysis**

Now recall the expression for the trace from equation (B.12):

$$\operatorname{tr} J(\mu, \phi) = \hat{z} + \gamma(2\theta - 1) + \gamma(1 - 3\theta)\,\mu \tag{B.20}$$

$$\operatorname{tr} J(\mu, \phi) = \hat{z} + \gamma(2\theta - 1) + \gamma(1 - 3\theta)\,\mu \tag{B.21}$$

Let $\mu_- = 1 - |\delta|$, $\mu_- - 1 = -|\delta|$, $|\delta| \geq 0$, $\hat{z} = -\gamma\theta\,|\delta|$ and define $\kappa(\theta) = \dfrac{\theta}{1-2\theta}$, plug into the trace and re-arrange so we get:

$$\begin{aligned}
\operatorname{tr} J(\mu_-, \phi) &= \hat{z} + \gamma(2\theta - 1) + \gamma(1 - 3\theta)\,\mu_- \\
&= -\,\gamma\theta|\delta| + \gamma(2\theta - 1) + \gamma(1 - 3\theta)\,(1 - |\delta|) \\
&= -\,\gamma\theta - \gamma(1 - 2\theta)\,|\delta| \\
&= \gamma\big[(2\theta - 1)\,|\delta| - \theta\big] \\
&= -\,\gamma(1 - 2\theta)\left(|\delta| + \frac{\theta}{1 - 2\theta}\right) \\
&= -\,\gamma(1 - 2\theta)\,\big(|\delta| + \kappa(\theta)\big)\,,
\end{aligned}$$

$$\boxed{\operatorname{tr} J(\mu_-, \phi) \;=\; -\,\gamma(1 - 2\theta)\,\big(|\delta| + \kappa(\theta)\big)} \tag{B.22}$$

$$\begin{aligned}
\det J(\mu_+, \phi) &= \theta(\theta - 1)\,\gamma^2\,|\delta|\,(1 + |\delta|), \\
\operatorname{tr} J(\mu_+, \phi) &= \gamma(1 - 2\theta)\,\big(|\delta| - \kappa(\theta)\big), \qquad \kappa(\theta) = \frac{\theta}{1 - 2\theta}, \\
\Delta_J(\mu_+, \phi) &= (\operatorname{tr} J)^2 - 4\det J = \gamma^2\,(|\delta| + \theta)^2 \;>\; 0;
\end{aligned}$$

$$
\begin{aligned}
\det J(\mu_-,\phi) &= \theta(1-\theta)\,\gamma^2\,|\delta|\,(1-|\delta|),\\
\operatorname{tr} J(\mu_-,\phi) &= -\,\gamma(1-2\theta)\,\big(|\delta|+\kappa(\theta)\big) = \gamma\big[(2\theta-1)|\delta|-\theta\big],\\
\Delta_J(\mu_-,\phi) &= (\operatorname{tr} J)^2 - 4\det J = \gamma^2\,(|\delta|-\theta)^2 \ \geq\ 0.
\end{aligned}
$$

$$
\begin{aligned}
\mu_+^* &= 1+|\delta| \quad &\Rightarrow& \quad |\delta| \in [0,\infty);\\
\mu_-^* &= 1-|\delta| \quad &\Rightarrow& \quad |\delta| \in (0,1).
\end{aligned}
$$

Under–mechanization $(\theta > 1)$:

$$
\begin{aligned}
\mu_+ : &\quad \det > 0,\ \operatorname{tr} < 0,\ \Delta > 0\ \Rightarrow\ \textbf{Stable node for all}\ |\delta| \geq 0,\\
\mu_- : &\quad \det < 0\ \Rightarrow\ \text{Saddle for all } |\delta| \in (0,1),\\
&\qquad \operatorname{tr} = 0 \text{ at } |\delta| = \frac{\theta}{2\theta-1} \in (0,1),\ \Delta > 0 \text{ there }\ \Rightarrow \text{hyperbolic saddle;}\\
&\qquad \det = 0 \text{ only at } |\delta| \in \{0,1\} \Rightarrow \text{non-hyperbolic edges (one eigenvalue } = 0).
\end{aligned}
$$

Over–mechanization $(0 < \theta < 1)$:

$$
\begin{aligned}
\mu_+ : &\quad \det < 0\ \Rightarrow\ \text{Saddle for all } |\delta| \geq 0 \quad \operatorname{tr} J \text{ sign: } \begin{cases} \text{if } 0 < \theta < \frac{1}{2}, & \operatorname{tr} \lesseqgtr 0 \text{ at } |\delta| = \kappa(\theta),\\ \text{if } \frac{1}{2} < \theta < 1, & \operatorname{tr} < 0\ \forall |\delta| \geq 0, \end{cases}\\
\mu_- : &\quad \det > 0,\ \operatorname{tr} < 0,\ \Delta \geq 0\ \Rightarrow\ \textbf{Stable node for all}\ |\delta| \in (0,1)\\
&\qquad \text{(degenerate stable node at } |\delta| = \theta \text{ since } \Delta = 0).
\end{aligned}
$$

C Harrodian Reversal

C.1 Derivation of the Total Derivative $\dfrac{d\hat{\phi}}{d\hat{u}}$

We start from the two-equation dynamic system:

$$\hat{\phi} = (\gamma u - 1)\hat{u} + (1-\theta)\gamma(u-1) \tag{C.1}$$
$$\hat{u} = \hat{z} + \Phi(\phi)\hat{\phi} - \theta\gamma(u-1) \tag{C.2}$$

We treat $\hat{\phi}$ as an implicit function of $\hat{u}$ through these equations and aim to derive the total derivative $\frac{d\hat{\phi}}{d\hat{u}}$.

C.1.1 Step 1: Differentiate Equation (1) totally w.r.t. $\hat{u}$

$$\begin{aligned}
\frac{d\hat{\phi}}{d\hat{u}} &= \frac{d}{d\hat{u}}\left[(\gamma u - 1)\hat{u} + (1-\theta)\gamma(u-1)\right] \\
&= (\gamma u - 1) + \hat{u}\frac{d}{d\hat{u}}(\gamma u) + (1-\theta)\gamma\frac{du}{d\hat{u}} \\
&= (\gamma u - 1) + \gamma\hat{u}\frac{du}{d\hat{u}} + (1-\theta)\gamma\frac{du}{d\hat{u}} \\
&= (\gamma u - 1) + \left[\gamma\hat{u} + (1-\theta)\gamma\right]\frac{du}{d\hat{u}}
\end{aligned}$$

$$\frac{d\hat{\phi}}{d\hat{u}} = (\gamma u - 1) + \left[\gamma\hat{u} + (1-\theta)\gamma\right]\frac{du}{d\hat{u}} \tag{C.3}$$

C.1.2 Step 2: Differentiate the dynamic equation for $\hat{u}$ with respect to $\hat{u}$

Solving for u equation (C.2) we get:

$$u = \frac{1}{\theta\gamma}\left(\hat{z} + \Phi(\phi)\hat{\phi} - \hat{u}\right) + 1$$

We now differentiate u with respect to $\hat{u}$, applying the chain rule to the term $\Phi(\phi)\hat{\phi}$, which depends on $\hat{u}$ through both ϕ and $\hat{\phi}$:

$$
\begin{aligned}
\frac{du}{d\hat{u}} &= \frac{d}{d\hat{u}}\left[\frac{1}{\theta\gamma}\left(\hat{z} + \Phi(\phi)\hat{\phi} - \hat{u}\right) + 1\right] \\
&= \frac{1}{\theta\gamma}\left(\frac{d}{d\hat{u}}[\Phi(\phi)\hat{\phi}] - 1\right)
\end{aligned}
$$

Applying the product rule:

$$
\frac{d}{d\hat{u}}[\Phi(\phi)\hat{\phi}] = \Phi'(\phi)\frac{d\phi}{d\hat{u}}\hat{\phi} + \Phi(\phi)\frac{d\hat{\phi}}{d\hat{u}}
$$

Therefore,

$$
\frac{du}{d\hat{u}} = \frac{1}{\theta\gamma}\left[\Phi'(\phi)\frac{d\phi}{d\hat{u}}\hat{\phi} + \Phi(\phi)\frac{d\hat{\phi}}{d\hat{u}} - 1\right] \tag{C.4}
$$

C.1.3 Step 3: Plug into the total derivative of $\frac{d\hat{\phi}}{d\hat{u}}$

Recalling the definition in the proposition,

$$
\Theta(\hat{u}, \theta) = \frac{\hat{u} + (1 - \theta)}{\theta}
$$

And returning to the expression for the total derivative of $\hat{\phi}$ with respect to $\hat{u}$ in equation (C.3) substituting the expression for $\frac{du}{d\hat{u}}$ found in equation (C.4) obtained in Step 2, then:

$$
\begin{aligned}
\frac{d\hat{\phi}}{d\hat{u}} &= (\gamma u - 1) + \Theta(\hat{u}, \theta)\left[\Phi'(\phi)\frac{d\phi}{d\hat{u}}\hat{\phi} + \Phi(\phi)\frac{d\hat{\phi}}{d\hat{u}} - 1\right] \\
&= (\gamma u - 1) - \Theta(\hat{u}, \theta) + \Theta(\hat{u}, \theta)\Phi(\phi)\frac{d\hat{\phi}}{d\hat{u}} + \Theta(\hat{u}, \theta)\Phi'(\phi)\frac{d\phi}{d\hat{u}}\hat{\phi} \\
[1 - \Theta(\hat{u}, \theta)\Phi(\phi)]\frac{d\hat{\phi}}{d\hat{u}} &= (\gamma u - 1) - \Theta(\hat{u}, \theta) + \Theta(\hat{u}, \theta)\Phi'(\phi)\frac{d\phi}{d\hat{u}}\hat{\phi}
\end{aligned}
$$

Therefore,

$$\frac{d\hat{\phi}}{d\hat{u}} = \frac{(\gamma u - 1) - \Theta(\hat{u},\theta) + \Theta(\hat{u},\theta)\Phi'(\phi)\dfrac{d\phi}{d\hat{u}}\hat{\phi}}{1 - \Theta(\hat{u},\theta)\Phi(\phi)} \tag{C.5}$$

Equation (C.5) carries a conundrum given the absence of an expression for $\dfrac{d\phi}{d\hat{u}}$, hence, the following step requires identifying it before solving the final expression for $\dfrac{d\hat{\phi}}{d\hat{u}}$.

Step 4: Finding $\dfrac{d\phi}{d\hat{u}}$

We begin with the structural expression for the investment share, derived from $\phi = I/Y$, which also can be expressed as $\phi = \dfrac{I}{K}\dfrac{K}{Y^p}\dfrac{Y^p}{Y}$. Hence, considering the dynamic accumulation $\dfrac{I}{K} = \gamma(u-1)$ and production functions $Y^p = K^\theta$ we get:

$$\phi = \frac{\gamma(u-1)\ K^{1-\theta}}{u}$$

Taking the total derivative of ϕ with respect to u, treating K as predetermined:

$$\frac{d\phi}{du} = \gamma K^{1-\theta}\frac{u-(u-1)}{u^2} = \frac{\gamma K^{1-\theta}}{u^2}$$

Now applying the chain rule:

$$\frac{d\phi}{d\hat{u}} = \frac{d\phi}{du}\frac{du}{d\hat{u}} = \frac{\gamma K^{1-\theta}}{u^2}\frac{du}{d\hat{u}}$$

Recall from Step 2 equation (C.4):

$$\frac{du}{d\hat{u}} = \frac{1}{\theta\gamma}\left[\Phi'(\phi)\frac{d\phi}{d\hat{u}}\hat{\phi} + \Phi(\phi)\frac{d\hat{\phi}}{d\hat{u}} - 1\right]$$

Substituting into the chain rule expression:

$$\begin{aligned}
\frac{d\phi}{d\hat{u}} &= \frac{\gamma K^{1-\theta}}{u^2}\left\{\frac{1}{\theta\gamma}\left[\Phi'(\phi)\frac{d\phi}{d\hat{u}}\hat{\phi} + \Phi(\phi)\frac{d\hat{\phi}}{d\hat{u}} - 1\right]\right\} \\
&= \frac{K^{1-\theta}}{\theta u^2}\left[\Phi'(\phi)\frac{d\phi}{d\hat{u}}\hat{\phi} + \Phi(\phi)\frac{d\hat{\phi}}{d\hat{u}} - 1\right]
\end{aligned}$$

$$\begin{aligned}
\frac{d\phi}{d\hat{u}} &= \frac{\cancel{\gamma} K^{1-\theta}}{u^2}\left\{\frac{1}{\theta\cancel{\gamma}}\left[\Phi'(\phi)\frac{d\phi}{d\hat{u}}\hat{\phi} + \Phi(\phi)\frac{d\hat{\phi}}{d\hat{u}} - 1\right]\right\} \\
&= \frac{K^{1-\theta}}{\theta u^2}\left[\Phi'(\phi)\frac{d\phi}{d\hat{u}}\hat{\phi} + \Phi(\phi)\frac{d\hat{\phi}}{d\hat{u}} - 1\right] \\
\Rightarrow \quad \frac{d\phi}{d\hat{u}} &= \frac{K^{1-\theta}}{\theta u^2}\Phi'(\phi)\frac{d\phi}{d\hat{u}}\hat{\phi} + \frac{K^{1-\theta}}{\theta u^2}\Phi(\phi)\frac{d\hat{\phi}}{d\hat{u}} - \frac{K^{1-\theta}}{\theta u^2} \\
\frac{d\phi}{d\hat{u}}\left(1 - \frac{K^{1-\theta}}{\theta u^2}\Phi'(\phi)\hat{\phi}\right) &= \frac{K^{1-\theta}}{\theta u^2}\left[\Phi(\phi)\frac{d\hat{\phi}}{d\hat{u}} - 1\right]
\end{aligned}$$

Hence,

$$\frac{d\phi}{d\hat{u}} = \frac{\dfrac{K^{1-\theta}}{\theta u^2}\left[\Phi(\phi)\dfrac{d\hat{\phi}}{d\hat{u}} - 1\right]}{1 - \dfrac{K^{1-\theta}}{\theta u^2}\Phi'(\phi)\hat{\phi}} \tag{C.6}$$

Which for convenience we define $\psi(\theta, u, K) = \dfrac{K^{1-\theta}}{\theta u^2}$ can express as:

$$\frac{d\phi}{d\hat{u}} = \frac{\psi(\theta, u, K)\left[\Phi(\phi)\dfrac{d\hat{\phi}}{d\hat{u}} - 1\right]}{1 - \psi(\theta, u, K)\Phi'(\phi)\hat{\phi}}$$

This yields an expression for $\dfrac{d\phi}{d\hat{u}}$.

C.1.4 Step 5: Solve for $\frac{d\hat{\phi}}{d\hat{u}}$

$$\frac{d\hat{\phi}}{d\hat{u}} = \frac{(\gamma u - 1) - \Theta + \dfrac{\Theta\psi\Phi'\hat{\phi}\Phi\dfrac{d\hat{\phi}}{d\hat{u}} - \Theta\psi\Phi'\hat{\phi}}{1 - \psi\Phi'\hat{\phi}}}{1 - \Theta\Phi}$$

$$= \frac{(\gamma u - 1) - \Theta}{1 - \Theta\Phi} + \frac{\Theta\psi\Phi'\hat{\phi}\Phi\dfrac{d\hat{\phi}}{d\hat{u}} - \Theta\psi\Phi'\hat{\phi}}{(1 - \Theta\Phi)(1 - \psi\Phi'\hat{\phi})}$$

$$= \frac{(\gamma u - 1) - \Theta}{1 - \Theta\Phi} - \frac{\Theta\psi\Phi'\hat{\phi}}{(1 - \Theta\Phi)(1 - \psi\Phi'\hat{\phi})} + \frac{\Theta\psi\Phi'\hat{\phi}\Phi}{(1 - \Theta\Phi)(1 - \psi\Phi'\hat{\phi})}\frac{d\hat{\phi}}{d\hat{u}}$$

$$\Rightarrow \quad \frac{d\hat{\phi}}{d\hat{u}}\left[1 - \frac{\Theta\psi\Phi'\hat{\phi}\Phi}{(1 - \Theta\Phi)(1 - \psi\Phi'\hat{\phi})}\right] = \frac{(\gamma u - 1) - \Theta}{1 - \Theta\Phi} - \frac{\Theta\psi\Phi'\hat{\phi}}{(1 - \Theta\Phi)(1 - \psi\Phi'\hat{\phi})}$$

$$\left[1 - \frac{\Theta\,\psi\,\Phi'\,\hat{\phi}\,\Phi}{(1 - \Theta\,\Phi)(1 - \psi\,\Phi'\,\hat{\phi})}\right] = \frac{(1 - \Theta\,\Phi)(1 - \psi\,\Phi'\,\hat{\phi})}{(1 - \Theta\,\Phi)(1 - \psi\,\Phi'\,\hat{\phi})} - \frac{\Theta\,\psi\,\Phi'\,\hat{\phi}\,\Phi}{(1 - \Theta\,\Phi)(1 - \psi\,\Phi'\,\hat{\phi})}$$

$$= \frac{(1 - \Theta\,\Phi)(1 - \psi\,\Phi'\,\hat{\phi}) - \Theta\,\psi\,\Phi'\,\hat{\phi}\,\Phi}{(1 - \Theta\,\Phi)(1 - \psi\,\Phi'\,\hat{\phi})}$$

$$= \frac{1 - \psi\,\Phi'\,\hat{\phi} - \Theta\,\Phi + \Theta\,\Phi\,\psi\,\Phi'\,\hat{\phi} - \Theta\,\psi\,\Phi'\,\hat{\phi}\,\Phi}{(1 - \Theta\,\Phi)(1 - \psi\,\Phi'\,\hat{\phi})}$$

$$= \frac{1 - \psi\,\Phi'\,\hat{\phi} - \Theta\,\Phi}{(1 - \Theta\,\Phi)(1 - \psi\,\Phi'\,\hat{\phi})}$$

$$\Rightarrow \quad \frac{d\hat{\phi}}{d\hat{u}}\frac{(1-\psi\,\Phi'\,\hat{\phi}-\Theta\,\Phi)}{(1-\Theta\,\Phi)(1-\psi\,\Phi'\,\hat{\phi})} = \frac{(\gamma u-1)-\Theta}{1-\Theta\Phi} - \frac{\Theta\psi\Phi'\hat{\phi}}{(1-\Theta\Phi)(1-\psi\Phi'\hat{\phi})}$$

$$\begin{aligned}
\frac{d\hat{\phi}}{d\hat{u}}(1-\psi\,\Phi'\,\hat{\phi}-\Theta\,\Phi) &= \cancel{(1-\Theta\,\Phi)}(1-\psi\,\Phi'\,\hat{\phi})\tfrac{(\gamma u-1)-\Theta}{\cancel{1-\Theta\Phi}} - \cancel{(1-\Theta\,\Phi)(1-\psi\,\Phi'\,\hat{\phi})}\tfrac{\Theta\psi\Phi'\hat{\phi}}{\cancel{(1-\Theta\Phi)(1-\psi\Phi'\hat{\phi})}} \\
&= (1-\psi\,\Phi'\,\hat{\phi})(\gamma u-1-\Theta) - \Theta\psi\Phi'\hat{\phi} \\
&= (1-\psi\,\Phi'\,\hat{\phi})(\gamma u-1) - (1-\psi\,\Phi'\,\hat{\phi})\Theta - \Theta\psi\Phi'\hat{\phi} \\
&= (1-\psi\,\Phi'\,\hat{\phi})(\gamma u-1) - \Theta\big[(1-\psi\,\Phi'\,\hat{\phi}) + \psi\Phi'\hat{\phi}\big] \\
&= (1-\psi\,\Phi'\,\hat{\phi})(\gamma u-1) - \Theta \\
&= (\gamma u-1)(1-\psi\Phi'\hat{\phi}) - \Theta
\end{aligned}$$

Such that the solution for $\dfrac{d\hat{\phi}}{d\hat{u}}$ can be expressed as:

$$\boxed{\frac{d\hat{\phi}}{d\hat{u}} = \frac{(\gamma u-1)(1-\psi\Phi'\hat{\phi})-\Theta}{1-\psi\,\Phi'\,\hat{\phi}-\Theta\,\Phi}} \tag{C.7}$$

C.2 Proposition and Proof of Harrodian Reversal in the MSM

Proposition C.1. *Let the dynamic behavior of the investment share $\hat{\phi}$ in the Marxist Supermultiplier (MSM) be described by the total derivative:*

$$\boxed{\frac{d\hat{\phi}}{d\hat{u}} = \frac{(\gamma u-1)(1-\psi\Phi'\hat{\phi})-\Theta}{1-\psi\,\Phi'\,\hat{\phi}-\Theta\,\Phi}} \tag{C.8}$$

where:

$$\begin{aligned}
\Theta(\hat{u},\theta) &= \frac{\hat{u}+1-\theta}{\theta}, \\
\psi(\theta,u,K) &= \frac{K^{1-\theta}}{\theta u^2}, \\
\Phi(\phi) &= \frac{\phi}{s-\phi+m}, \qquad \Phi'(\phi) = \frac{s+m}{(s-\phi+m)^2}.
\end{aligned}$$

Then the system exhibits a Harrodian reversal, i.e. a contraction in the investment share of output $\hat{\phi}$ despite rising rate of capacity utilization $\hat{u}$ — if and only if:

$$\frac{d\hat{\phi}}{d\hat{u}} < 0 \quad \Longleftrightarrow \quad (\mathcal{N} > 0 \ \wedge \ \mathcal{D} < 0) \ \cup \ (\mathcal{N} < 0 \ \wedge \ \mathcal{D} > 0).$$

Proof. We begin with the total derivative of $\hat{\phi}$ with respect to $\hat{u}$:

$$\frac{d\hat{\phi}}{d\hat{u}} = \frac{(\gamma u - 1)(1 - \psi\Phi'\hat{\phi}) - \Theta}{1 - \psi\Phi'\hat{\phi} - \Theta\Phi}.$$

Let:

$$\begin{aligned} \mathcal{N} &= (\gamma u - 1)(1 - \psi\Phi'\hat{\phi}) - \Theta, \\ \mathcal{D} &= 1 - \psi\Phi'\hat{\phi} - \Theta\Phi. \end{aligned}$$

Then:

$$\frac{d\hat{\phi}}{d\hat{u}} < 0 \quad \Longleftrightarrow \quad \mathrm{sgn}(\mathcal{N}) \cdot \mathrm{sgn}(\mathcal{D}) < 0.$$

We examine this product in four possible configurations:

(i) *Overheating reversal:*
If utilization is high so that $(\gamma u - 1)(1 - \psi\Phi'\hat{\phi}) > \Theta$ (numerator positive), a reversal occurs if $\mathcal{D} < 0$, i.e. $\Theta\Phi > 1 - \psi\Phi'\hat{\phi}$.

(ii) *Overheating reversal with overshooting responsiveness:*
If $\psi\Phi'\hat{\phi} > 1$, the term $(1 - \psi\Phi'\hat{\phi})$ is negative and the numerator can still be positive depending on $\gamma u - 1$. In this case reversal also requires $\mathcal{D} < 0$, i.e. $\Theta\Phi > 1 - \psi\Phi'\hat{\phi}$.

(iii) *Recovery reversal:*
If utilization is low such that $(\gamma u - 1)(1 - \psi\Phi'\hat{\phi}) < \Theta$ (numerator negative), a reversal occurs if $\mathcal{D} > 0$, i.e. $\Theta\Phi < 1 - \psi\Phi'\hat{\phi}$.

(iv) *Recovery reversal with overshooting responsiveness:*
With $\psi\Phi'\hat{\phi} > 1$, the numerator can be negative even for higher utilization. In this case reversal occurs if $\mathcal{D} > 0$, i.e. $\Theta\Phi < 1 - \psi\Phi'\hat{\phi}$.

Thus, the slope is negative whenever numerator and denominator differ in sign, covering overheating and recovery phases under both moderate and overshooting responsiveness. ■

Proposition C.2 (Harrodian reversal at steady state)**.** *At the steady state $(\hat{u}, \hat{\phi}) = (0, 0)$ with utilization $u = u^*$ and investment share $\phi = \phi^*$, the slope of the Harrodian relation reduces to*

$$\left.\frac{d\hat{\phi}}{d\hat{u}}\right|_{ss} = \frac{(\gamma u^* - 1) - \Theta^*}{1 - \Theta^* \, \Phi(\phi^*)}, \qquad \Theta^* = \frac{1 - \theta}{\theta}.$$

A Harrodian reversal occurs at steady state if and only if numerator and denominator differ in sign, i.e.

$$\Big[(\gamma u^* - 1) - \Theta^*\Big]\Big[1 - \Theta^* \, \Phi(\phi^*)\Big] < 0.$$

Proof. We start from the general slope expression established in equation (C.8):

$$\frac{d\hat{\phi}}{d\hat{u}} = \frac{(\gamma u - 1)(1 - \psi \Phi'(\phi)\hat{\phi}) - \Theta}{1 - \psi \Phi'(\phi)\hat{\phi} - \Theta \Phi(\phi)}.$$

At the steady state $(\hat{u}, \hat{\phi}) = (0, 0)$ the term $\psi \, \Phi'(\phi)\hat{\phi}$ vanishes identically, so the numerator and denominator simplify to

$$\mathcal{N} = (\gamma u^* - 1) - \Theta, \qquad \mathcal{D} = 1 - \Theta \, \Phi(\phi^*).$$

Moreover, by definition $\Theta(\hat{u}, \theta) = \frac{\hat{u} + 1 - \theta}{\theta}$, which at $\hat{u} = 0$ becomes

$$\Theta^* = \frac{1 - \theta}{\theta}.$$

Substituting these into the slope gives

$$\left.\frac{d\hat{\phi}}{d\hat{u}}\right|_{\mathrm{ss}} = \frac{(\gamma u^* - 1) - \Theta^*}{1 - \Theta^* \, \Phi(\phi^*)}.$$

By inspection, the sign of this slope depends solely on the signs of the numerator and denominator. The slope is negative precisely when they differ in sign, that is,

$$\Big[(\gamma u^* - 1) - \Theta^*\Big]\Big[1 - \Theta^* \, \Phi(\phi^*)\Big] < 0.$$

This condition characterizes the Harrodian reversal at steady state: capacity utilization increasing while the investment share contracts. ■

D Important Partitions

Definition D.1 (Feasible parameter *states* by $\mathcal{G}$ and $\mathcal{Z}_{\text{admissible}}$)**.** Let $|\delta| = |\hat{z}|/(\gamma\theta)$ and let $\mathcal{Z}_{\text{admissible}}(\theta, \gamma)$ be as in Definition **??**. Define the sign–sliced admissible demand sets

$$\mathcal{Z}^{-}_{\text{adm}}(\theta, \gamma) = \mathcal{Z}_{\text{admissible}}(\theta, \gamma) \cap (-\infty, 0), \qquad \mathcal{Z}^{+}_{\text{adm}}(\theta, \gamma) = \mathcal{Z}_{\text{admissible}}(\theta, \gamma) \cap (0, \infty).$$

The *feasible parameter states* are

$$\begin{aligned} \mathcal{P}_{-} &= \Big\{(\theta, \gamma, \hat{z}) :\ \theta > 0,\ \hat{z} \in \mathcal{Z}^{-}_{\text{adm}}(\theta, \gamma),\ |\delta| \in (0, 1)\Big\} \\ \mathcal{P}_{+} &= \Big\{(\theta, \gamma, \hat{z}) :\ \theta > 0,\ \hat{z} \in \mathcal{Z}^{+}_{\text{adm}}(\theta, \gamma),\ |\delta| > 0\Big\} \end{aligned}$$

Intersecting with the investment regimes $\mathcal{G}$ gives

$$\mathcal{P}^{\text{mild}}_{\pm} = \mathcal{P}_{\pm} \cap \{\gamma \in \mathcal{G}_{\text{mild}}\}, \qquad \mathcal{P}^{\text{strong}}_{\pm} = \mathcal{P}_{\pm} \cap \{\gamma \in \mathcal{G}_{\text{strong}}\}.$$

Equivalently, in $|\delta|$–coordinates:

$$\gamma \in \mathcal{G}_{\text{mild}} : \begin{cases} \mathcal{P}^{\text{mild}}_{-} :\ |\delta| \in (0, 1), \\ \mathcal{P}^{\text{mild}}_{+} :\ |\delta| > 0, \end{cases} \qquad \gamma \in \mathcal{G}_{\text{strong}} : \begin{cases} \mathcal{P}^{\text{strong}}_{-} :\ |\delta| \in \big(\lambda^{-}_{\text{crit}}(\theta, \gamma),\ 1\big), \\ \mathcal{P}^{\text{strong}}_{+} :\ |\delta| \in \big(\lambda^{+}_{\text{crit}}(\theta, \gamma),\ \infty\big), \end{cases}$$

where $\lambda^{-}_{\text{crit}} = |r_1|/(\gamma\theta)$ and $\lambda^{+}_{\text{crit}} = r_2/(\gamma\theta)$ are induced by the roots $r_1 < r_2$ of $\Delta(\hat{z}) = 0$ (Def. **??**). The upper bound $|\delta| < 1$ in $\mathcal{P}_{-}$ enforces $u^{*}_{-} = 1 - |\delta| > 0$; endpoints $r_{1,2}$ remain admissible in $\mathcal{Z}_{\text{admissible}}$ but are excluded here when they violate feasibility.

Definition D.2 (Accumulation Regimes defined by Mechanization)**.** Tag each feasible point by

$$\Theta_{\text{over}} = \{0 < \theta < 1\}, \qquad \Theta_{\text{under}} = \{\theta > 1\},$$

and define regime-sliced feasible sets by intersection, e.g. $\mathcal{P}^{\text{OM}} = \mathcal{P} \cap \{\theta \in \Theta_{\text{over}}\}$ and $\mathcal{P}^{\text{UM}} = \mathcal{P} \cap \{\theta \in \Theta_{\text{under}}\}$. The θ-cut shifts r_1, r_2 (hence $\lambda^{\pm}_{\text{crit}}$) but does not change the γ-piecewise structure.