Distributive Cycles, Financial Fragility, and Wealth Inequality in a Goodwin–Minsky Model

Diego Polanco

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Abstract

This paper develops a stepped modeling sequence to clarify how endogenous distributive and financial cycles can generate persistent pressure toward asset inequality, even when aggregate macro–financial dynamics remain bounded. The analysis begins from the canonical growth-cycle model of Goodwin [1967], in which distributive conflict between employment and the wage share produces locally neutral closed orbits around an interior fixed point. It then introduces a Goodwin–Minsky extension in the spirit of Minsky [1986] and Keen [1995, 2013], adding debt as a stock variable and a bounded, nonlinear investment function. This mutation breaks the closed-orbit genericity of the Goodwin model and yields a three-dimensional macro–financial system whose local dynamics may converge, sustain oscillations, or amplify into instability, depending on institutional and financial parameters.

Building on this core, the paper re-centers the analysis on asset inequality by developing a constrained finance-augmented Goodwin–Minsky system. Without expanding the state space beyond employment, the wage share, and leverage, the model introduces an explicit financial outside option and a profitability-dependent portfolio wedge that generates endogenous financial pressure and asset-income claims. This construction preserves the Hopf-based regime classification of the baseline model while altering its economic interpretation: the same local stability boundaries that separate macro regimes also partition regimes by the persistence of financial pressure and the conditions under which asset-income claims systematically outpace macro growth.

The contribution of the paper is analytical. It provides a transparent identification device linking endogenous macro–financial regimes to asset-inequality pressure, prior to the introduction of a full stock–flow consistent wealth-accumulation sector. By comparing the Jacobian structure and Hopf bifurcation logic of the baseline and finance-augmented systems, the paper establishes a foundation for subsequent work that maps these regimes into explicit wealth dynamics under accounting consistency.

1 Introduction

A recurring tension in contemporary capitalist economies is that aggregate dynamics can appear bounded or even stabilizing, while asset concentration continues to rise as a cumulative outcome. Employment, output growth, and leverage may fluctuate within admissible ranges for long periods, yet the distribution of asset claims shifts persistently in favor of wealth holders. This paper develops a stepped modeling sequence designed to make that tension analytically explicit. The goal is not to claim that wealth concentration has no feedback on macroeconomic outcomes in reality, but to construct an *identification device*: a transparent sequence of closures that separates (i) the stability and bifurcation logic of a low-dimensional macro–financial core from (ii) the mechanisms through which those dynamics generate sustained pressure toward asset inequality.

The point of departure is the canonical growth-cycle model of Goodwin [1967]. The Goodwin system formalizes distributive conflict as a two-dimensional interaction between the employment rate and the wage share. Under its tight closure, the model yields closed orbits around an interior fixed point: endogenous cycles arise without exogenous shocks, but their amplitudes are not selected by dissipative forces internal to the model. This local neutrality is precisely what makes the Goodwin model a useful benchmark. It isolates a conflict-driven mechanism linking functional income distribution to profit-led accumulation and, through employment, to trend-plus-cycle movements in living standards.

A large literature has examined the empirical and theoretical status of Goodwin-type dynamics. Empirical contributions emphasize the sensitivity of estimated cycles to functional forms, parameter stability, and measurement conventions [Araújo et al., 2019]. Related theoretical work shows that once finance is introduced, trajectories may resemble Goodwin oscillations while the underlying mechanism is no longer the canonical two-dimensional center [Stockhammer and Michell, 2017]. These results motivate a structural approach: treat the Goodwin cycle as the baseline mechanism, then relax closures one at a time so that changes in qualitative dynamics can be attributed to specific additions rather than to undisciplined complexity.

The first closure change is financial. Following the Goodwin–Minsky tradition in the spirit of Minsky [1986] and the Keen-style formulations [Keen, 1995, 2013], the paper introduces debt as a stock variable linking current accumulation decisions to future financial obligations. Investment becomes a bounded, nonlinear function of net profitability, breaking the mechanical identity between contemporaneous profits and accumulation. Once debt enters, instability is no longer a matter of initial conditions around a neutral center. Depending on parameters governing investment responsiveness and leverage-contingent wage discipline, the resulting three-dimensional system can converge to an interior steady state, sustain bounded oscillations, or evolve toward amplifying macro–financial cycles. The classification of these regimes is governed by the local geometry of the system, diagnosed through the Jacobian, Routh–Hurwitz conditions, and a Hopf functional reported in Appendix A.

The second step of the paper re-centers the analysis on asset inequality *before* introducing a full wealth-accumulation sector. The Goodwin–Minsky core captures endogenous macro–financial instability, but by itself it does not identify how cycles, stabilization, or breakdown translate into persistent asymmetries in asset claims. To address this gap, Section 4 develops a constrained Goodwin–Minsky–finance core that preserves the three-dimensional state space (e, ω, d) while embedding an explicit financial outside option. An exogenous benchmark return r_F and a profitability-dependent portfolio wedge generate an endogenous financial-income channel and a financial-pressure index that enters wage discipline. This construction does not add new state variables, but it changes the economic interpretation of the Jacobian and the Hopf bifurcation: the same local stability boundary that classifies macro regimes also partitions regimes by the implied persistence of financial pressure and the conditions under which asset-income claims systematically outpace macro growth.

The contribution of this constrained core is analytical. It shows how asset inequality can be treated as an induced consequence of endogenous distributive–financial cycles, their amplification into instability, or their collapse into stable stagnation, rather than as an exogenous trend. By keeping the state space fixed, the model allows a direct comparison between the Hopf structure of the baseline Goodwin–Minsky system and the finance-augmented core, clarifying what changes when financial outside options are made explicit. The full Jacobian structure and bifurcation conditions for this constrained core are reported in Appendix B.

The paper is intentionally incomplete with respect to wealth accumulation itself. The constrained core isolates the financial-return mechanism that generates persistent asset-income pressure, but it does not yet introduce a stock–flow consistent wealth sector with explicit asset prices and heterogeneous balance sheets. That extension is the next step of the project and the motivation for ongoing simulation work. The role of the present paper is to provide the analytical backbone: a clean mapping from endogenous macro–financial regimes to the conditions under which asset inequality can emerge and persist.

The remainder of the paper is organized as follows. Section 2 presents the canonical Goodwin growth-cycle model. Section 3 introduces the Goodwin–Minsky extension with debt dynamics and documents its local stability properties. Section 4 develops the constrained asset-inequality core and compares its Jacobian and Hopf interpretation to the baseline model. The conclusion summarizes the identification logic and outlines the next steps toward a full stock–flow consistent treatment of wealth inequality.

2 The Goodwin Model

The growth-cycle model introduced by Goodwin [1967] provides the canonical baseline for formalizing distributive conflict in a growing capitalist economy. The benchmark isolates a minimal interaction between employment and functional income distribution that is sufficient to generate endogenous cyclical dynamics under a closed accumulation rule. In this paper, thGoodwin system serves as the reference structure against which subsequent extensions with finance, debt dynamics, and wealth accumulation are assessed.

Production is characterized by a fixed-coefficients technology. Output is constrained by capital and effective labour in fixed proportions,

$$Y(t) = \min\left\{\frac{K(t)}{\sigma},\, a(t)L(t)\right\}, \tag{1}$$

where $\sigma > 0$ denotes the capital–output ratio and $a(t)$ labour productivity. Labour productivity and the labour force grow at constant exogenous rates,

$$\frac{\dot{a}}{a} = \alpha, \tag{2}$$

$$\frac{\dot{N}}{N} = \beta, \tag{3}$$

with $\alpha, \beta > 0$. Capital accumulation is profit-determined: all profits are invested and the profit share governs the rate of capital growth. Finally, real wage growth responds positively to labour-market tightness through a Phillips-curve mechanism.

The dynamics are governed by two state variables. The employment rate is

$$e \equiv \frac{L}{N}, \tag{4}$$

and the wage share is

$$\omega \equiv \frac{W}{Y}, \qquad W \equiv wL, \tag{5}$$

with normalized prices $P = 1$ and w the real wage. The profit share is $\pi \equiv 1 - \omega$.

Under the assumptions above, output and capital grow at the profit-determined rate,

$$g_Y \equiv \frac{\dot{Y}}{Y} = \frac{\dot{K}}{K} = \frac{\pi}{\sigma} = \frac{1-\omega}{\sigma}. \tag{6}$$

Employment dynamics follow from comparing profit-led accumulation growth to the exogenous growth of effective labour supply. The reduced-form evolution of the employment rate is

$$\dot{e} = \left(\frac{1-\omega}{\sigma} - (\alpha + \beta) \right) e. \tag{7}$$

Real wage growth follows a Phillips curve,

$$\frac{\dot{w}}{w} = \Phi(e), \qquad \Phi'(e) > 0, \tag{8}$$

which implies wage-share dynamics

$$\dot{\omega} = \big[\Phi(e) - \alpha\big]\omega. \tag{9}$$

Equations (15)–(16) define the canonical Goodwin system.

An interior stationary point (e^*, ω^*) satisfies

$$\Phi(e^*) = \alpha, \tag{10}$$

$$\frac{1-\omega^*}{\sigma} = \alpha + \beta. \tag{11}$$

Hence

$$\omega^* = 1 - \sigma(\alpha + \beta), \tag{12}$$

and e^* is pinned down by the wage-setting schedule via (10). Provided $0 < e^* < 1$ and $0 < \omega^* < 1$, the fixed point is economically admissible.

Linearization around (e^*, ω^*) yields a center: the Jacobian has zero trace and positive determinant, implying purely imaginary eigenvalues. As a result, the system generates closed orbits in (e, ω) space. Cycle amplitudes are not selected by parameters; they depend on initial conditions. Figure 1 illustrates this neutrality: trajectories form nested closed cycles around the interior fixed point.

The canonical cycle admits a transparent mapping from distributive conflict to accumulation and living standards. When employment is high, wage growth accelerates via $\Phi(e)$, raising the wage share and compressing profits. Lower profits reduce the accumulation rate (14), which in turn reduces employment growth through (15). As employment falls, wage pressure weakens, the wage share declines, profitability recovers, and accumulation strengthens. This feedback produces persistent oscillations in both employment and functional distribution.

Figure 2 complements the phase diagram by translating the cycle into macroeconomic observables. The wage share $\omega(t)$ oscillates around ω^* and, through (14), generates corresponding oscillations in the profit-led accumulation rate $g_Y(t)$. To relate the cycle to living standards, note that under the Leontief full-capacity branch $Y = aL$, so output per capita satisfies

$$\frac{Y}{N} = a\,e. \tag{13}$$

Since $a(t)$ grows exogenously at rate α while $e(t)$ oscillates endogenously, output per capita displays a trend increase with cyclical modulation, as shown in the bottom panel of Figure 2.

As a benchmark, the canonical Goodwin model is intentionally restricted. It abstracts from debt and financial stocks, from explicit investment behaviour beyond the profit-investment closure, from demand constraints, and from household balance-sheet heterogeneity. These exclusions are design choices that isolate the distributive mechanism and generate neutral cycles around an interior fixed point. Subsequent sections relax these closures, first by introducing debt-finance dynamics, opening the model to Minsky understanding of financial instability, as first introduceed by Keen [1995]. Later, through a set of stock-flow consistent system of equations, the model is further extended to study conditions under which distributive cycles can be associated with endogenous dynamics of wealth inequality .

Figure 1: Canonical Goodwin phase dynamics: closed orbits around (e^*, ω^*).

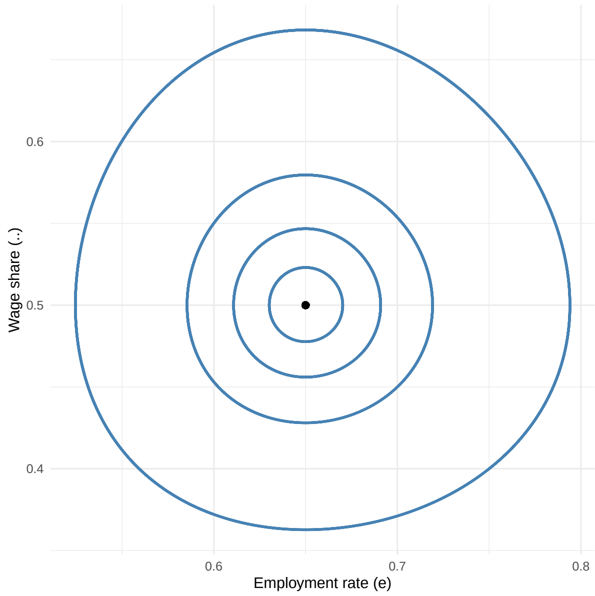
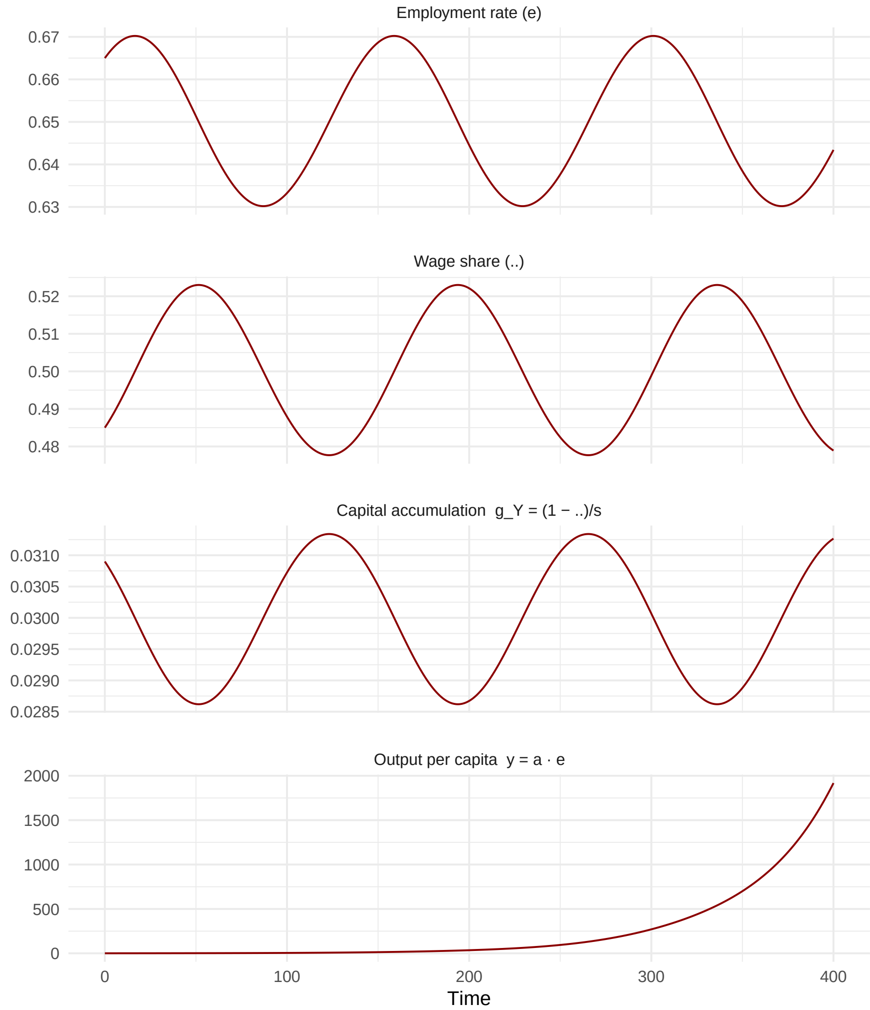


Figure 2: Distribution, accumulation, and output per capita in a mild canonical cycle.



3 Financial Instability: The Goodwin–Minsky model

The canonical Goodwin cycle is a two-dimensional benchmark that organizes capitalist dynamics around a specific closure: profits are fully re-invested, while capital accumulation creates productive capacities through a fixed-coefficients production function without mismatches of demand and supply. When that closure is relaxed, distributive dynamics are no longer insulated from balance-sheet commitments. The Goodwin–Minsky models first introduced by Keen [1995, 2013] exploits this shift in closure by introducing debt as a stock variable that links current accumulation decisions to future financial obligations aiming to identify dynamcis of financial stability *a-la* Minsky [1986]. The resulting models preserve the labor-market mechanism of the Goodwin cycle while moving beyond its flow-only structure of the system of state variables: instability is not imposed through exogenous shocks or ad hoc nonlinearities, but emerges endogenously from stock–flow interactions between profits, investment, and debt.

With fixed coefficients, a constant capital–output ratio $\sigma > 0$, and the full-investment-of-profits closure, output growth in the canonical Goodwin model is

$$\frac{\dot{Y}}{Y} \equiv g_Y = \frac{1-\omega}{\sigma}. \tag{14}$$

Let $g_n \equiv \alpha + \beta$ denote the exogenous growth rate of effective labor supply (productivity plus labor-force growth). The Goodwin dynamics for employment e and wage share ω can be written as

$$\dot{e} = \left(\frac{1-\omega}{\sigma} - g_n\right) e, \tag{15}$$

$$\dot{\omega} = \big[\Phi(e) - \alpha\big]\omega. \tag{16}$$

This two-dimensional system implies a closed-orbit property around the interior stationary point: trajectories near that point are closed, and cycle amplitude is determined by initial conditions rather than by dissipative forces internal to the model.

The Goodwin–Minsky mutation begins by breaking the mechanical identity between contemporaneous profits and accumulation. Investment is specified as a share of output that depends positively on profitability. In the Keen-style specification adopted here, the investment share $\kappa(\,\cdot)$ is bounded and nonlinear:

$$\kappa(r) = \kappa_{\min} + \frac{\kappa_{\max} - \kappa_{\min}}{1 + \exp\{-\lambda(r - r_0)\}}, \qquad \kappa_{\min} < \kappa(r) < \kappa_{\max}, \qquad \lambda > 0. \tag{17}$$

With a depreciation rate $\delta \geq 0$, and fixed capital-output ratio σ output growth becomes defined by:

$$\frac{\dot{Y}}{Y} \equiv g_Y(r) = \frac{\kappa(r)}{\sigma} - \delta. \tag{18}$$

This modification preserves the distributive structure of the Goodwin model while breaking its flow-only accumulation logic: accumulation is no longer mechanically constrained to equal profits.

Allowing accumulation to deviate from contemporaneous profits requires external finance. Let d denote a debt-to-output ratio. The profit share remains

$$\pi(\omega) \equiv 1 - \omega, \tag{19}$$

but the profitability object variable for accumulation is the net profit rate, which is reduced by the interest burden on debt:

$$r(\omega, d) = \frac{\pi(\omega) - id}{\sigma} = \frac{1 - \omega - id}{\sigma}, \qquad i \geq 0. \tag{20}$$

Debt evolves by a stock–flow accounting law that closes the financial side of the model:

$$\dot{d} = r(\omega, d)\big) - \pi(\omega) + id - d\, g_Y\big(r(\omega, d)\big) \tag{21}$$

Debt introduces an intertemporal asymmetry absent from the Goodwin baseline: wages and profits are flow variables, while d accumulates over time and feeds back through interest obligations and slow-downs in economic growth. In this sense, the system shifts along a complexity vector: the labor-market mechanism is retained, while balance-sheet commitments become a co-determining state.

To preserve the Goodwin labor-market mechanism while allowing institutional feedback from leverage, I introduce a a debt-contingent term into the Phillips curve defining a leverage threshold $\bar{d} > 0$, normalized to 1, and a logistic function which imposes a smooth switch in a similar fashion to the Buffet index of stock-value to output:

$$S(d) = \frac{1}{1 + \exp\{-\psi\,(d-1)\}}, \qquad \psi > 0, \qquad 0 < S(d) < 1. \tag{22}$$

Wage growth is then specified as

$$\Phi(e, d) = \phi_0 + \phi_1 e - \eta S(d), \qquad \phi_1 > 0, \qquad \eta \geq 0. \tag{23}$$

The interpretation is that tight labor markets raise wage pressure through $\phi_1 e$, while $d < 1$ has a positive effect on wage growth, and $d > 1$ a negative one. This switching mechanism in the model can be interpreted as loose space to acquire debt and pump-up demand, but too high financial obligations discipline it through demand disciplined mechanisms (such as fiscal consolidation or rising inflation due debt-commitments in foreign currency). Overall, the interpretation is that debt as a share of output growth might reinforce or discipline wage share growth through demand channels. The parameters η and ψ govern the strength of this institutional restraint around $d \simeq \bar{d}$ normalized to 1 in this formulation.

I combine the modified accumulation rule, the labor-market mechanism, and the debt stock law, which yields a three-dimensional dynamical system in employment, distribution, and leverage:

$$\dot{e} = \big(g_Y(r) - g_n\big)e, \tag{24}$$

$$\dot{\omega} = \big(\Phi(e, d) - \alpha\big)\omega, \tag{25}$$

$$\dot{d} = \kappa(r) - \pi(\omega) + id - d\, g_Y(r), \tag{26}$$

where $r = r(\omega, d)$ is given by (20), $g_Y(r)$ by (18), and $\Phi(e, d)$ by (23). Relative to the Goodwin benchmark, the mutation is minimal in form but maximal in implication: the distributive mechanism is retained, yet the phase portrait is no longer confined to the two-dimensional closed-orbit property.

Introducing debt alters the qualitative behavior of the system. The model is no longer characterized by a closed-orbit property as the generic local outcome. Instead, depending on parameter values, trajectories may converge to an interior stationary point, exhibit persistent oscillations, or diverge toward explosive leverage dynamics. Instability arises endogenously from the interaction of distributive conflict and balance-sheet dynamics: debt transforms contemporaneous profit–investment relations into intertemporal commitments, and net profitability transmits financial fragility into accumulation. The discipline mechanism can counteract or reshape this instability by altering the wage-share feedback once leverage approaches the threshold.

For analytical transparency, the full local stability conditions (Jacobian structure, Routh–Hurwitz inequalities, and Hopf functional) are reported in Appendix A. In this Keen-ish 3D structure, the Hopf bifurcation theorem allows to identify the bifurcation space as follows

$$H = E(FD + CB), \tag{27}$$

Where:

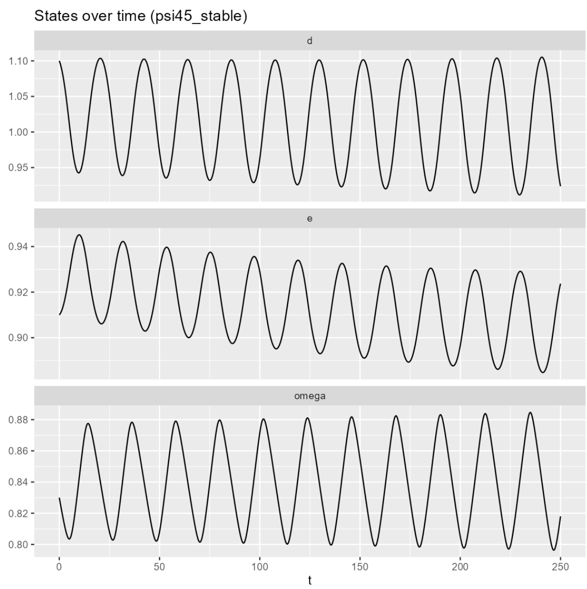
$$A = -\, e^* \,\frac{\kappa_r(r^*)}{\sigma^2}, \qquad B = -\, e^* \,\frac{i\,\kappa_r(r^*)}{\sigma^2}, \tag{28}$$

$$C = \omega^* \,\phi_1, \qquad D = -\,\omega^* \,\eta\, S_d(d^*), \tag{29}$$

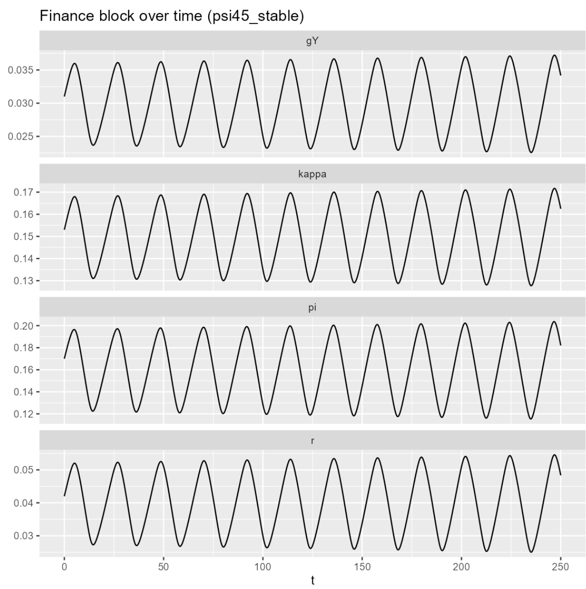
$$E = 1 - \left(1 - \frac{d^*}{\sigma}\right)\frac{\kappa_r(r^*)}{\sigma}, \qquad F = (i - g_n) - \left(1 - \frac{d^*}{\sigma}\right)\frac{i\,\kappa_r(r^*)}{\sigma}. \tag{30}$$

Figure 3 illustrates a bounded cyclical regime generated by the stock–flow interaction between net profitability and accumulation: (e, ω, d) remain in the admissible region while $\kappa(r)$, $g_Y(r)$, and $r(\omega, d)$ oscillate around their balanced-growth values. The leverage-triggered discipline term shifts the wage-share feedback when d approaches $\bar{d}$, preventing explosive leverage paths in this parameterization. The corresponding $y_{pc}(t)$ series inherits its dominant trend from $\exp(\alpha t)$, with cyclical modulation through $e(t)$.

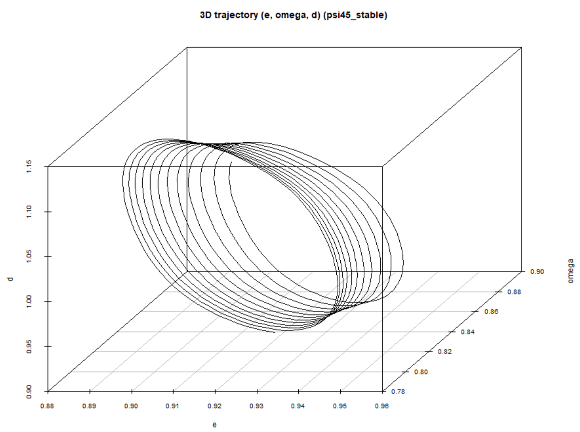
Figure 3: Stable cyclical regime under leverage-triggered discipline ($\psi = 45$). The state variables remain bounded in the admissible region, and the finance block oscillates around the balanced-growth anchor.



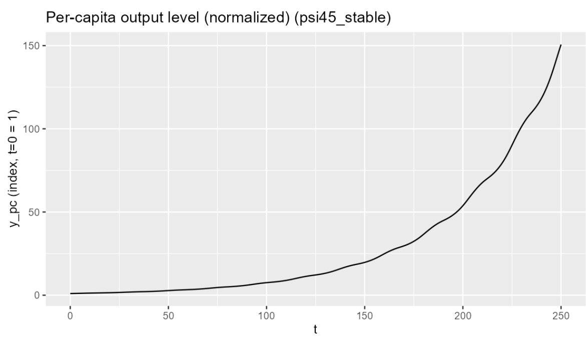
(a) States: $d(t)$, $e(t)$, $\omega(t)$.



(b) Finance block: $g_Y(t)$, $\kappa(t)$, $\pi(t)$, $r(t)$.



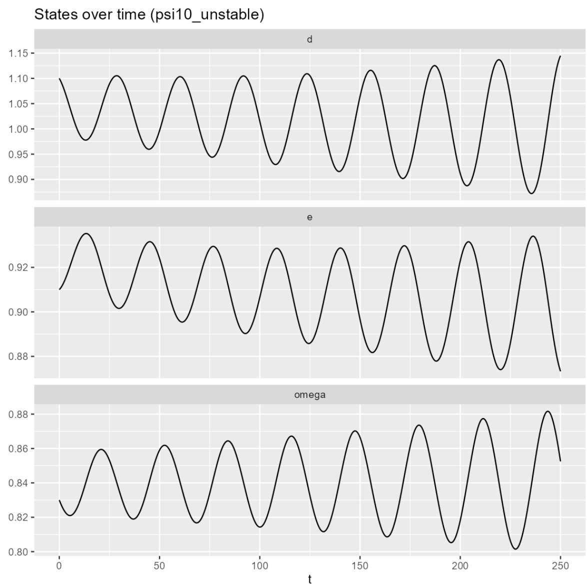
(c) 3D trajectory in (e, ω, d).



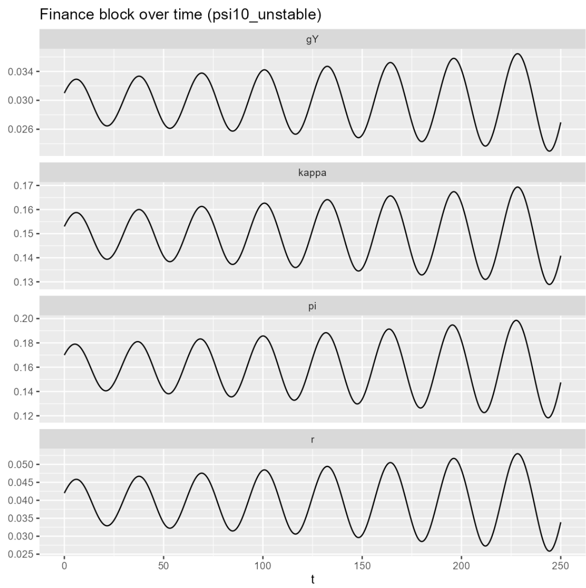
(d) Per-capita output level $y_{pc}(t)$ (normalized).

Figure 4 illustrates a regime in which the interior steady state is not attracting: starting from a small perturbation, the trajectory moves outward and the oscillations in (e, ω, d) amplify over time. The finance block provides the transmission channel: widening fluctuations in net profitability $r(\omega, d)$ induce wider movements in $\kappa(r)$ and therefore in $g_Y(r)$, feeding back into employment dynamics through $\dot{e} = (g_Y - g_n)e$ and into leverage dynamics through $\dot{d} = \kappa(r) - \pi(\omega) + id - dg_Y(r)$. In this parameterization, lower discipline steepness ψ weakens the leverage-triggered restraint in $\Phi(e, d)$, so leverage does not settle to an interior level and the system evolves toward higher-amplitude debt–distribution–employment cycles within the simulation horizon.

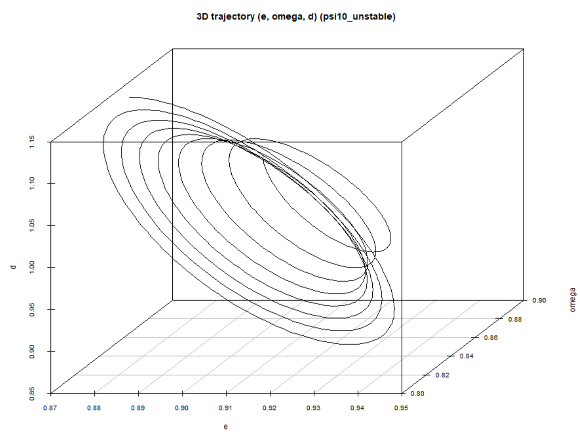
Figure 4: Amplifying cycle regime with weak leverage-triggered discipline ($\psi = 10$). The trajectory moves away from the interior steady state and evolves toward higher-amplitude debt–distribution–employment oscillations within the simulation horizon.



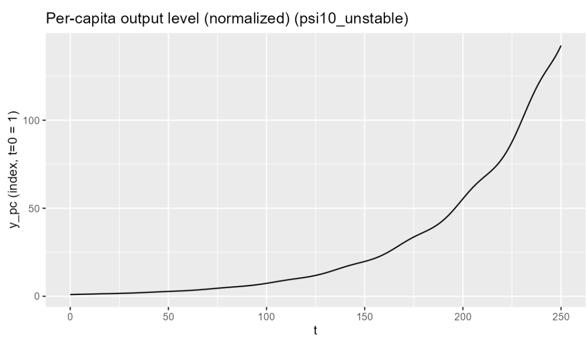
(a) States: $d(t)$, $e(t)$, $\omega(t)$.



(b) Finance block: $g_Y(t)$, $\kappa(t)$, $\pi(t)$, $r(t)$.



(c) 3D trajectory in (e, ω, d).



(d) Per-capita output level $y_{pc}(t)$ (normalized).

Despite its richer dynamics, Keen's Goodwin–Minsky model remains limited in two respects. First, households are treated homogeneously, and distributive conflict is summarized exclusively by the wage share. Second, while debt affects macroeconomic stability through net profitability and interest obligations, the model does not track asset ownership, wealth accumulation, or heterogeneous balance-sheets for stock-flow consistency.

The Goodwin–Minsky framework demonstrates that once debt is introduced as a stock variable, distributive cycles are no longer dynamically neutral. Financial commitments transform the Goodwin system into a dissipative one, allowing for amplification, instability, and crisis. This extension succeeds in explaining why capitalist economies may exhibit endogenous financial fragility without relying in exogenous shocks.

However, the scope of the Goodwin–Minsky model remains limited in a crucial respect. Although debt redistributes income over time through interest payments and affects macroeconomic stability, households are treated as a homogeneous sector. Distributional conflict is summarized exclusively by the wage share, and the ownership of assets and liabilities is not modeled explicitly. As a result, the framework cannot distinguish between instability in aggregate dynamics and divergence in household wealth positions.

This limitation is not merely descriptive. Financial instability and wealth inequality are analytically distinct phenomena. An economy may experience rising leverage and cyclical instability without generating persistent divergence in household net worth, just as wealth inequality may increase in the absence of overt financial crises. The Goodwin–Minsky framework captures the former but remains silent on the latter.

Section III addresses this gap by introducing two types of households: asset owners and non-asset owners, and wealth dynamics while preserving the distributive and financial mechanisms in (24)–(26) from the Goodwin–Minsky model. The objective is not to generalize the framework, but to construct a specific model in which wealth inequality emerges endogenously as a consequence of distributive cycles, debt accumulation, and asset ownership. By treating wealth inequality as a driven accumulation outcome rather than an exogenous trend or steady-state property, the analysis connects financial instability to systematic divergence in household wealth under distinct growth regimes, rather than only at moments of crisis.

In this way, Section III builds directly on the mechanisms developed in Sections I and II, extending them only where necessary to explain a phenomenon that the existing framework cannot account for on its own

4 Asset Inequality as an Induced Outcome of Endogenous Cycles under Stock-Flow Consistent Constraints

The Goodwin–Minsky model developed in the previous section establishes how distributive conflict and debt dynamics can generate endogenous cycles, convergence, or instability through a three-dimensional stock–flow system. That framework is sufficient to classify macro–financial regimes via local stability and Hopf bifurcation analysis. It is not, however, sufficient to track the accumulation of asset inequality as a *cumulative outcome* of those regimes.

This section introduces a constrained analytical core whose purpose is to bridge that gap. The objective is not to add wealth accumulation directly, but to isolate the financial-return mechanism through which endogenous macro cycles, their amplification, or their stabilization generate persistent asymmetries in asset claims. These asymmetries will form the backbone of the stock–flow consistent wealth-inequality block introduced in the next section.

The strategy is deliberate. By restricting the state space to three dimensions while embedding a financial outside option, the model preserves the Hopf bifurcation logic of the Goodwin–Minsky system while re-centering its interpretation: cycles and regime shifts are no longer only about employment and distribution, but about the *conditions under which asset income persistently outpaces the growth of the economy.*

4.1 From macro–financial cycles to asset inequality

The state vector remains

$$x(t) = (e(t), \omega(t), d(t)),$$

with employment e, wage share ω, and debt-to-output ratio d. Net profitability is

$$r(\omega, d) = \frac{1 - \omega - id}{\sigma}, \tag{31}$$

and accumulation follows the bounded investment rule

$$g(r) = \frac{\kappa(r)}{\sigma} - \delta, \tag{32}$$

with $\kappa(r)$ defined as in the previous section.

The key extension relative to the baseline Goodwin–Minsky model is the introduction of an *external financial return benchmark* r_F. This benchmark does not enter accumulation directly. Instead, it governs the allocation of profit claims between productive accumulation and financial income through the logistic portfolio-share function

$$\lambda(r, r_F) = \frac{1}{1 + \exp\{-\psi(r - r_F)\}}, \qquad \psi > 0. \tag{33}$$

Financial income is then defined as

$$\iota_F = r\,\frac{\lambda}{1 - \lambda}, \tag{34}$$

and normalized by the natural growth rate to obtain the financial pressure index

$$f = \frac{\iota_F}{g_n}. \tag{35}$$

The interpretation is straightforward. When net profitability approaches or exceeds the financial benchmark r_F, a rising share of profit claims is diverted toward financial income. This does not alter the accumulation rule mechanically, but it reshapes the distributional environment in which accumulation takes place. In particular, it creates a wedge between output growth and the growth of asset claims, a necessary condition for persistent asset inequality.

4.2 Financial discipline and wage dynamics

Financial conditions discipline the labor market through the composite index

$$Z(d, f) = \frac{1}{1 + \exp\{\phi_3[(d - 1) + \phi_4(f - 1)]\}}, \tag{36}$$

which rises with leverage and financial pressure. Wage dynamics follow

$$\dot{\omega} = \omega\big(\phi_0 + \phi_1 e - \alpha - \phi_2 Z(d, f)\big), \tag{37}$$

with $\phi_1 > 0$ fixed exogenously.

Relative to the Goodwin–Minsky model of the previous section, this specification shifts the role of finance. Debt no longer disciplines wages only through interest burdens and growth slowdowns, but also through a financial outside option whose strength depends on the gap between productive and financial returns. The consequence is that macro–financial cycles can generate sustained periods in which financial claims grow faster than output even when employment and the wage share remain bounded.

4.3 Core dynamics and interior steady state

The reduced system is

$$\dot{e} = (g(r) - g_n)e, \tag{38}$$

$$\dot{\omega} = \omega(\phi_0 + \phi_1 e - \alpha - \phi_2 Z), \tag{39}$$

$$\dot{d} = \kappa(r) - (1 - \omega) + id - dg(r). \tag{40}$$

An interior steady state satisfies $g(r^*) = g_n$, implying

$$\kappa(r^*) = \sigma(g_n + \delta). \tag{41}$$

The associated steady-state values of d^*, ω^*, and e^* follow from accounting identities and the wage-setting condition, as in the previous section.

Crucially, the existence of an interior steady state in (e, ω, d) does not imply neutrality of asset income. Even at a macro–financial steady state, the financial pressure index f^* and the discipline term $Z(d^*, f^*)$ can remain strictly positive, sustaining a persistent gap between financial and productive income flows.

4.4 Jacobian structure and comparison with the Goodwin–Minsky model

Linearization around the interior steady state yields a 3×3 Jacobian that is formally similar to that of the Goodwin–Minsky model. The system remains dissipative, and local dynamics are classified by the Routh–Hurwitz conditions and a Hopf bifurcation theorem.

The crucial difference lies in the composition of the Jacobian. In the baseline Goodwin–Minsky model, instability is driven by feedbacks between profitability, accumulation, and debt. In the constrained core developed here, those feedbacks remain, but they are augmented by derivatives of the financial allocation function $\lambda(r, r_F)$ and the pressure index f. As a result, the Hopf boundary now partitions regimes not only by macro stability, but by the implied dynamics of financial claims.

A Hopf bifurcation in this reduced system therefore has a dual interpretation. As in the previous section, it marks the transition between convergent, cyclical, and unstable macro dynamics. In addition, it identifies thresholds beyond which endogenous cycles generate sustained financial pressure, laying the groundwork for persistent asset inequality even in regimes that appear macro-stable.

The full Jacobian, derivative structure, and Hopf functional are reported in Appendix B. The key implication is structural: asset inequality does not require explosive macro dynamics. It can emerge as a cumulative outcome of bounded cycles or of stabilization paths that nevertheless preserve a positive financial outside option.

5 Conclusion

This paper developed a stepped modeling sequence to clarify a simple but stubborn tension: capitalist economies can exhibit bounded macro–financial dynamics while the distribution of asset claims shifts persistently in favor of wealth holders. The objective was not to deny that wealth concentration can feed back into macro outcomes in reality, but to construct an identification device: a transparent set of closures that separates (i) the stability and bifurcation logic of a low-dimensional macro–financial core from (ii) the distributional mechanisms that translate those dynamics into persistent asset-inequality pressure.

The sequence proceeds in three steps. Section 2 established the canonical benchmark: the Goodwin [Goodwin, 1967] growth-cycle model, where distributive conflict between employment and the wage share generates locally neutral closed orbits around an interior fixed point. The analytical value of this benchmark is precisely its restriction. It isolates the conflict-driven feedback from employment to wages, from wages to profits, and from profits to accumulation, while abstracting from balance sheets and financial commitments.

Section 3 introduced the Goodwin–Minsky mutation in the spirit of Minsky [1986] and Keen [1995, 2013]. Debt enters as a stock variable and investment becomes a bounded nonlinear function of net profitability, breaking the Goodwin model's generic closed-orbit property. The resulting three-dimensional system can converge, sustain bounded oscillations, or move toward amplifying macro–financial cycles depending on institutional and financial parameters. This regime classification is governed by local geometry: the Jacobian, Routh–Hurwitz conditions, and the Hopf functional reported in Appendix A.

Section 4 re-centered the analysis on the distributional object of interest before introducing a full wealth-accumulation sector. The section developed a constrained asset-inequality core that preserves the three-dimensional state space (e, ω, d) but embeds an exogenous benchmark financial return r_F and a profitability-dependent portfolio wedge $\lambda(r, r_F)$. This wedge generates an endogenous financial-income channel and a financial-pressure index f that enters wage discipline through the composite $Z(d, f)$. The key contribution of this step is interpretive: it keeps the Hopf-based regime logic of the Goodwin–Minsky system, while changing what the same stability boundary *means*. In the baseline model, a Hopf boundary partitions regimes by macro stability (convergence versus sustained cycles versus instability). In the asset-inequality core, the same boundary additionally partitions regimes by the implied persistence of financial pressure and the conditions under which asset-income claims systematically outpace macro growth, even when the macro state remains bounded. The full Jacobian structure and Hopf conditions for this constrained core are therefore placed in Appendix B.

Two implications follow. First, asset inequality can be treated as an induced consequence of endogenous distributive–financial cycles and their regime shifts, rather than as an exogenous trend appended after the macro analysis. Second, the relevant bifurcation logic is not only about whether the macro–financial system converges or cycles, but about whether the regime implied by the same local geometry sustains a positive financial outside option and persistent financial pressure.

The paper is intentionally incomplete in one important respect, and that incompleteness is the point of the research design. The constrained core isolates the financial-return mechanism while holding the state space fixed; it does not yet introduce a full stock–flow consistent wealth-accumulation block with asset prices and heterogeneous balance sheets. That block is the next step and the purpose of the ongoing simulation work: to map the macro–financial regimes identified here into explicit wealth trajectories and inequality dynamics while preserving accounting discipline. Once that extension is in place, the stepped architecture developed in this draft will provide a clean bridge between (i) regime classification in the macro–financial core and (ii) the cumulative distributional outcomes that follow from it.

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A Goodwin-Minsky: Model specification and Stability Analysis

A.1 State vector, domains, and exogenous growth

The state vector is $x(t) = (e(t), \omega(t), d(t))$, where the employment rate satisfies $e(t) \in (0, 1)$, the wage share satisfies $\omega(t) \in (0, 1)$, and the debt-to-output ratio satisfies $d(t) > 0$. Exogenous growth rates are labor productivity growth $\alpha > 0$ and labor-force growth $\beta \geq 0$, implying the natural growth rate

$$g_n \equiv \alpha + \beta > 0. \tag{42}$$

A.2 Auxiliary definitions: profits, net profitability, investment, output growth

Define the profit share as

$$\pi(\omega) \equiv 1 - \omega. \tag{43}$$

Let $\sigma > 0$ denote the (constant) capital–output ratio and $i \geq 0$ the interest rate. Net profitability is summarized by the net profit rate

$$r(\omega, d) = \frac{\pi(\omega) - id}{\sigma} = \frac{1 - \omega - id}{\sigma}. \tag{44}$$

Investment is specified as a bounded share of output, $\kappa(r) \in (\kappa_{\min}, \kappa_{\max})$, with a logistic Keen form

$$\kappa(r) = \kappa_{\min} + \frac{\kappa_{\max} - \kappa_{\min}}{1 + \exp\{-\lambda(r - r_0)\}}, \qquad \lambda > 0. \tag{45}$$

Let $\delta \geq 0$ denote depreciation. Capacity-branch output growth is

$$g_Y(r) = \frac{\kappa(r)}{\sigma} - \delta. \tag{46}$$

A.3 Debt leverage threshold and wage restraint)

Let $\bar{d} > 0$ denote the leverage threshold and define the leverage index $B(d) \equiv d/\bar{d}$. The smooth discipline switch is

$$S(d) = \frac{1}{1 + \exp\{-\psi(d - \bar{d})\}} = \frac{1}{1 + \exp\{-\psi(d - 1)\}}, \qquad \psi > 0, \qquad 0 < S(d) < 1. \tag{47}$$

Wage inflation is specified by a disciplined Phillips term

$$\Phi(e, d) = \phi_0 + \phi_1 e - \eta S(d), \qquad \phi_1 > 0, \qquad \eta \geq 0. \tag{48}$$

A.4 Core dynamics (3D system)

The ODE system is

$$\dot{e} = \big(g_Y(r) - g_n\big)e, \tag{49}$$

$$\dot{\omega} = \big(\Phi(e, d) - \alpha\big)\omega, \tag{50}$$

$$\dot{d} = \kappa(r) - \pi(\omega) + id - d\, g_Y(r), \tag{51}$$

with $r = r(\omega, d)$ from (44) and $g_Y(r)$ from (46).

A.5 Slow-finance option (time-scale separation)

An optional slow-finance parameter $\tau_d \geq 1$ rescales debt adjustment:

$$\dot{d} = \frac{1}{\tau_d}\Big(\kappa(r) - \pi(\omega) + id - d\, g_Y(r)\Big). \tag{52}$$

This affects adjustment speeds but does not alter the interior steady state.

A.6 Interior steady state and admissibility gates (closed form)

An interior steady state (e^*, ω^*, d^*) satisfies $\dot{e} = \dot{\omega} = \dot{d} = 0$ with $e^* \in (0,1)$, $\omega^* \in (0,1)$, $d^* > 0$. From $\dot{e} = 0$, the balanced-growth condition implies

$$g_Y(r^*) = g_n \quad \Longleftrightarrow \quad \kappa(r^*) = \sigma(g_n + \delta) \equiv \kappa_{\text{target}}. \tag{53}$$

Gate 1 (investment feasibility):

$$\kappa_{\min} < \kappa_{\text{target}} < \kappa_{\max}. \tag{54}$$

Define

$$s \equiv \frac{\kappa_{\text{target}} - \kappa_{\min}}{\kappa_{\max} - \kappa_{\min}} \in (0,1). \tag{55}$$

Logistic inversion yields the steady-state net profit rate

$$r^* = r_0 + \frac{1}{\lambda} \log\left(\frac{s}{1-s}\right). \tag{56}$$

Using $\kappa(r^*) = \kappa_{\text{target}}$ and (44), the steady-state debt ratio is

$$d^* = \frac{\kappa_{\text{target}} - \sigma r^*}{g_n}. \tag{57}$$

Gate 2 (interior leverage): $d^* > 0$.
From $\dot{d} = 0$ and $g_Y(r^*) = g_n$,

$$0 = \kappa_{\text{target}} - \pi(\omega^*) + i d^* - d^* g_n, \tag{58}$$

$$\pi(\omega^*) = \kappa_{\text{target}} + d^*(i - g_n), \tag{59}$$

$$\omega^* = 1 - \kappa_{\text{target}} - d^*(i - g_n). \tag{60}$$

Gate 3 (interior distribution): $0 < \omega^* < 1$.
From $\dot{\omega} = 0$, $\Phi(e^*, d^*) = \alpha$, so

$$e^* = \frac{\alpha - \phi_0 + \eta S(d^*)}{\phi_1}. \tag{61}$$

Gate 4 (interior employment): $0 < e^* < 1$. (Optional plausibility screening can restrict e^* to an interval without affecting algebra.)

A.7 Jacobian at the steady state (sparse form and components)

Let J denote the Jacobian of $(\dot{e}, \dot{\omega}, \dot{d})$ evaluated at (e^*, ω^*, d^*). The Jacobian has the sparse structure

$$J = \begin{bmatrix} 0 & A & B \\ C & 0 & D \\ 0 & E & F \end{bmatrix}. \tag{62}$$

Define the derivatives

$$\kappa_r(r) \equiv \frac{\partial \kappa}{\partial r} = (\kappa_{\max} - \kappa_{\min})\lambda\, \ell(r)\big(1 - \ell(r)\big), \qquad \ell(r) \equiv \frac{1}{1 + \exp\{-\lambda(r - r_0)\}}, \tag{63}$$

$$S_d(d) \equiv \frac{\partial S}{\partial d} = \frac{\psi}{\bar{d}}\, S(d)\big(1 - S(d)\big). \tag{64}$$

Evaluated at (e^*, ω^*, d^*), the Jacobian components are

$$A = -\,e^* \,\frac{\kappa_r(r^*)}{\sigma^2}, \qquad B = -\,e^* \,\frac{i\,\kappa_r(r^*)}{\sigma^2}, \tag{65}$$

$$C = \omega^*\, \phi_1, \qquad D = -\,\omega^*\, \eta\, S_d(d^*), \tag{66}$$

$$E = 1 - \left(1 - \frac{d^*}{\sigma}\right) \frac{\kappa_r(r^*)}{\sigma}, \qquad F = (i - g_n) - \left(1 - \frac{d^*}{\sigma}\right) \frac{i\,\kappa_r(r^*)}{\sigma}. \tag{67}$$

A.8 Routh–Hurwitz conditions and Hopf functional

Let the characteristic polynomial be

$$\det(sI - J) = s^3 + a_1 s^2 + a_2 s + a_3. \tag{68}$$

For the sparse Jacobian (62), the coefficients are

$$a_1 = -F, \qquad a_2 = -AC - ED, \qquad a_3 = C(AF - BE). \tag{69}$$

Local asymptotic stability requires the Routh–Hurwitz inequalities

$$a_1 > 0, \quad a_2 > 0, \quad a_3 > 0, \quad a_1 a_2 > a_3. \tag{70}$$

Define the Hopf functional

$$H \equiv a_1 a_2 - a_3. \tag{71}$$

In the sparse structure (62), H admits the identity

$$H = E(FD + CB). \tag{72}$$

A Hopf boundary occurs at $H = 0$ subject to the remaining sign restrictions in (70).

A.9 Hopf transversality (finite-difference check)

For a bifurcation parameter $\mu \in \{\psi, \eta, \lambda, i, \dots\}$, transversality can be assessed by recomputing the steady state and $H(\mu)$ for perturbed values and forming the symmetric finite difference

$$\frac{dH}{d\mu} \approx \frac{H(\mu + h) - H(\mu - h)}{2h}, \tag{73}$$

where each $H(\mu \pm h)$ is computed after re-solving (e^*, ω^*, d^*) via the admissible steady-state construction above.

A.10 Simulation outputs and normalization used in figures

Numerical simulation integrates (49)–(51) over a grid $t \in [0, T]$ with step Δt. Per-capita output is reported as an index

$$y_{pc}(t) = a(t)e(t), \qquad a(t) = \exp(\alpha t), \qquad y_{pc}^{\text{norm}}(t) = \frac{y_{pc}(t)}{y_{pc}(0)}. \tag{74}$$

For each simulation tag, the standard figure set comprises: state trajectories (e, ω, d), finance-block series (r, π, κ, g_Y), normalized y_{pc}, and the 3D trajectory in (e, ω, d).

B Analytical Details: Jacobian and Hopf Conditions for the Asset-Inequality Core

This appendix reports the analytical details underlying the reduced core introduced in Section **??**. The Jacobian is obtained by explicit differentiation of $r(\omega, d)$, $\kappa(r)$, $\lambda(r, r_F)$, and $Z(d, f)$, accounting for the dependence of financial pressure f on profitability.

The characteristic polynomial takes the form

$$\det(\lambda I - J) = \lambda^3 + a_1 \lambda^2 + a_2 \lambda + a_3,$$

with coefficients a_1, a_2, a_3 reported explicitly in terms of structural parameters and steady- state values. Local stability requires the Routh–Hurwitz conditions

$$a_1 > 0, \qquad a_2 > 0, \qquad a_3 > 0, \qquad a_1 a_2 > a_3.$$

A Hopf bifurcation occurs when

$$H = a_1 a_2 - a_3 = 0,$$

holding the remaining sign restrictions fixed. Relative to the Goodwin–Minsky model of Appendix A, the expression for H includes additional terms proportional to the derivatives of $\lambda(r, r_F)$ and $Z(d, f)$, reflecting the role of the financial outside option. These terms do not alter the dimensionality of the system, but they reorient the economic interpretation of the bifurcation toward asset-income dynamics.