Growth, Distribution, and Working Class Power

Diego Polanco

January 4, 2026

Abstract

This paper develops a dynamic account of working-class power using a Goodwin [1967] model of growth and distribution. Rather than treating power as an external institutional correction to market outcomes, the paper conceptualizes power as a relational magnitude that is simultaneously an outcome of economic conditions and a determinant of their evolution. In the canonical Goodwin model, the employment rate and the wage share are percentage variables whose interaction generates distributive cycles. We show that the interior fixed point of this system can be interpreted as a baseline configuration of employment and distribution implied by a given institutional–technical regime, while the amplitude and persistence of cycles reflect the intensity and reproduction of distributive conflict.

Building on this interpretation, the paper distinguishes between structural power, rooted in labor market tightness, and associational power, rooted in organization, coordination, and solidarity. Structural power is already present in the canonical model through employment. Associational power is introduced as an institutional lever that conditions how tightness translates into wage dynamics, by shifting the intercept, slope, and local stability properties of the wage-setting relation. This disciplined one-equation extension preserves the accumulation block while allowing organizational variables to reshape distributive transmission.

The framework naturally yields a regime interpretation based on local stability. Using the Hopf bifurcation theorem, the paper shows how changes in associational power can move the system between damped adjustment and persistent distributive cycles, formalizing institutional transitions as stability shifts rather than level effects alone. The model is operationalized empirically using long-run Chilean data on wage shares, employment, and union structure, including union affiliation, fragmentation, and average union size. Preliminary evidence suggests that organizational configuration conditions distributive dynamics more strongly than affiliation alone, consistent with weak transmission from labor market tightness to wages in Chile.

The paper contributes to labor studies by providing a dynamic, institutionally grounded framework for analyzing growth, distribution, and working-class power, and by offering a formal but accessible language for studying how organization shapes the temporal structure of distributive conflict.

1 Introduction

Growth and distribution are often studied as outcomes of institutional arrangements, policy regimes, or macroeconomic constraints. Less often are they treated as part of an internally dynamic process in which conflict over income shares feeds back into the very conditions that make such conflict possible. In much of the labor studies literature, power appears as an external force: something that intervenes in wage setting, moderates inequality, or corrects market outcomes. In contrast, Goodwin-type growth–distribution models offer a different starting point. They formalize class conflict as a dynamic relation in which employment conditions and income distribution co-evolve over time, generating recurrent patterns rather than static equilibria.

This paper takes that dynamic insight seriously, but departs from the canonical Goodwin framework in one crucial respect. Rather than treating distributive conflict as an abstract struggle between capital and labor, we ask how working-class power—understood as a historically specific and institutionally mediated capacity—shapes the dynamics of growth and distribution. The central claim is simple: if power is relational, then it must be represented not only as an outcome of economic conditions, but also as a determinant of how those conditions translate into distributive change. In a growth–distribution cycle, power is both cause and consequence.

The Goodwin model is particularly well suited for this task because its state variables—the employment rate and the wage share—are themselves shares. They are percentages that encode relations of magnitude: how tight the labor market is, and how output is divided. The interior fixed point of the model therefore has a natural interpretation as a baseline configuration of employment and distribution implied by a given institutional–technical regime. Changes in bargaining institutions, labor market organization, or surplus allocation do not merely shift outcomes around that baseline; they reshape the conditions under which distributive conflict stabilizes, amplifies, or dissipates.

Building on this insight, the paper develops a disciplined strategy for introducing working-class power into a Goodwin framework. We distinguish between structural power, rooted in labor market tightness and absorption, and associational power, rooted in organization, coordination, and solidarity. Structural power is already present in the canonical model through employment. Associational power, however, is not. Introducing it requires an explicit institutional lever that conditions how tightness translates into wage pressure.

Rather than dispersing institutional effects across the entire model, we proceed incrementally. The first step is to institutionalize the wage-setting relation—the Phillips or bargaining curve—by allowing its intercept, slope, and local stability properties to depend on measures of working-class organization. This approach has three advantages. Conceptually, it aligns with how unions and collective action operate in practice, primarily through wage bargaining and enforcement. Analytically, it preserves the accumulation block, allowing us to isolate distributive mechanisms without conflating channels. Empirically, it yields testable implications for thresholds, amplitudes, and regime changes in observed wage–employment dynamics.

A key contribution of the paper is to show that once associational power is introduced in this way, the Goodwin system naturally lends itself to a regime interpretation based on local stability. In particular, the Hopf bifurcation theorem provides a precise language for describing how institutional configurations can shift the system from damped adjustment to persistent distributive cycles. In this reading, oscillations are not technical curiosities but expressions of sustained conflict, and changes in their amplitude and persistence correspond to changes in the balance of power.

The paper develops this argument in four steps. First, it revisits the canonical Goodwin model and interprets its fixed point and local geometry in terms of relations of magnitude that encode power. Second, it introduces a typology of working-class power and maps its components to specific elements of the wage-setting block. Third, it extends the model to allow organizational variables to act as institutional levers that shift stability properties, emphasizing the distinction between local dynamics and global bounding mechanisms. Finally, it operationalizes these ideas empirically using long-run Chilean data on wage shares, employment, and union structure.

Chile provides a particularly instructive case. Despite substantial variation in growth and employment over the twentieth century, wage-share gains have often failed to materialize or persist, even in periods of relatively tight labor markets. This pattern suggests not merely weak growth, but weak transmission from tightness to distribution—a diagnosis that points directly to associational power. By embedding this diagnosis in a dynamic growth–distribution framework, the paper aims to bridge labor studies and political economy, offering a formal but accessible account of how organization shapes the temporal structure of distributive conflict.

2 Labor power as cause and outcome in distributive cycles

Goodwin-type distributive cycle models are useful precisely because they treat class conflict as a *dynamic relation* rather than as a comparative-static wedge. In the canonical growth–distribution cycle, the wage share is simultaneously (i) an *outcome* of conflict and (ii) a *cause* of future macrodynamics. The wage share ω records the realized balance of claims on output at a point in time, but it also feeds back into accumulation and labor demand through profitability, thereby reshaping the employment rate e that conditions subsequent bargaining. In this sense, power is relational not only as a sociological premise but as a structural property of the model's feedback architecture. [Goodwin, 1967, CITATION NEEDED] This section develops that premise and motivates a disciplined strategy for introducing working-class power through a typology of its exercise, with a minimal first step that modifies only the wage-setting block. [, CITATION NEEDED]

2.1 Baseline Goodwin mechanics and the power loop

We take the baseline state variables as employment rate $e \equiv L/N$ and wage share $\omega \equiv W/Y$. A canonical Goodwin closure can be written as a two-dimensional system in which employment grows when accumulation outpaces effective labor-supply growth, while the wage share grows when wage pressure exceeds labor-productivity growth. For concreteness, we use a linear Phillips/bargaining mapping with intercept γ and slope ρ:

$$\dot{\omega} \;=\; \omega\big(\gamma + \rho e - a\big), \tag{1}$$

where a denotes labor-productivity growth, $a \equiv \dot{\alpha}/\alpha$ with $\alpha \equiv Y/L$. The employment-rate dynamic is driven by accumulation and effective labor-supply expansion:

$$\dot{e} \;=\; e\big(g(\omega) - n\big), \tag{2}$$

where $n \equiv \dot{N}/N$ is the growth of the labor force (or effective labor supply).

To connect the accumulation term to technique in a transparent way, we adopt a constant capital productivity convention $\sigma \equiv Y/K$ so that $Y = \sigma K$. With investment financed out of profits according to $s_\pi = 1 - c_\pi$ and profit share $(1-\omega)$, the implied accumulation term can be expressed as $s_\pi \sigma(1-\omega)$ net of depreciation and trend growth components. A convenient reduced form is

$$\dot{e} \;=\; e\Big((1-c_\pi)\sigma(1-\omega) - \delta - a - n\Big), \tag{3}$$

where δ is (economic) depreciation. This representation makes clear that distribution affects growth not only through profits $(1-\omega)$ but also through capitalists' surplus-allocation choice c_π (consumption out of profits), which scales the fraction of profits that becomes accumulation demand. In other words, distributive conflict shapes the conditions of employment growth, and employment growth in turn reshapes distributive conflict through (1). This is the basic power loop.

Finally, the constant σ convention is best interpreted as fixing a *technique regime* rather than as asserting a purely technical constant. Define mechanization as $\beta \equiv K/L$. Then

$$\sigma \equiv \frac{Y}{K} = \frac{\alpha}{\beta}, \tag{4}$$

and the associated growth-rate identity is

$$\frac{\dot{\sigma}}{\sigma} = a - b, \tag{5}$$

where $b \equiv \dot{\beta}/\beta$ is mechanization growth. Thus, holding σ constant corresponds to imposing $a = b$, i.e. a particular co-movement between labor productivity and mechanization. This matters because choices over mechanization, work organization, and the pace of restructuring are themselves sites of conflict within the labor process. The baseline model abstracts from these conflicts to isolate the distribution–employment feedback, but the interpretation of σ should remain explicitly institutional.

2.2 A typology of working-class power and a mapping to Goodwin blocks

If power is relational, it must be *exercised* through distinct channels. For the purposes of a Goodwin framework, it is useful to separate at least two forms that are both conceptually defensible and modelable with minimal additional structure:

(A) Structural power. Structural power refers to workers' leverage derived from labor-market tightness and the costs of replacement and disruption. In the baseline system, structural power is already represented by $e = L/N$ and its effect on wage pressure through ρ in (1). However, because e is a ratio, its meaning depends on both labor absorption (L) and the evolution of labor supply (N). If n is treated as endogenous or semi-endogenous (migration, participation responses, demographic dynamics, informal absorption), then e reflects a joint outcome of accumulation and social reproduction, and the slope ρ is best read as a reduced-form sensitivity that implicitly bundles these processes.

(B) Associational power. Associational power refers to workers' capacity for collective action and coordination (membership, resources, strike capacity, bargaining infrastructure). Union affiliation is a natural empirical proxy for this dimension. Importantly, associational power is not well represented by e itself: it is better conceived as shaping how a given level of tightness translates into wage outcomes. In a reduced-form Goodwin setting, this means that associational power should enter the wage-setting block as a shifter of the Phillips/bargaining mapping (intercept and/or slope), rather than as a direct driver of employment dynamics.

This typology provides a disciplined rationale for focusing first on the wage-share equation. Structural power is already present in the baseline model through tightness. The analytical task is therefore to introduce associational power in a way that does not confound channels: the simplest first step is to let union affiliation condition the wage-pressure mapping in (1), while holding the accumulation block (3) fixed. This isolates the effect of organized labor on the translation of tightness into distribution, before adding additional hypotheses about how unions affect investment behavior, the choice of technique, or the evolution of labor supply.

2.3 Why begin with a one-equation institutionalization of the Phillips curve

There are three reasons to begin by institutionalizing only the Phillips/bargaining relation.

First, it is the most direct locus of union influence: unions operate through wage bargaining, contract coverage, strike threats, and enforcement, all of which primarily affect wage-setting outcomes at given labor market conditions. [, CITATION NEEDED]

Second, methodological minimality improves interpretability. Because the Goodwin cycle is already a feedback system, allowing union affiliation to shift both wage dynamics and accumulation dynamics at once can obscure which channel generates changes in stability and cycle geometry. A one-equation extension provides a clean baseline for later, richer extensions.

Third, the Chilean case motivates precisely this wedge between tightness and wage outcomes. Chile has exhibited prolonged periods in which growth and labor-demand conditions have not translated into broad-based wage-share gains, consistent with institutional bottlenecks in wage transmission. [, CITATION NEEDED] A Goodwin-plus-associational-power extension aims to represent such bottlenecks formally by allowing organized labor to condition the wage-pressure mapping, thereby clarifying when a distributive cycle is damped, delayed, or skewed by weak associational capacity. This is the theoretical starting point for the paper's subsequent model extension and simulation analysis. [, CITATION NEEDED]

3 The interior fixed point, cycle amplitude, and the magnitude relations of power

If the previous section argued that power is relational because it feeds back on itself, the present section adds a second claim that matters for a labour-studies readership: power is also *scalar*. It is exercised in relations of magnitude, in percentage points, and through thresholds that separate "a tight labour market" from a politically effective one, or "a high wage share" from one that triggers a counter-movement in growth. Because both state variables in the Goodwin model are shares—the employment rate e and the wage share ω—the fixed point can be read as a pair of baseline percentages implied by a given growth–distribution regime, while the cycle amplitude can be read as the *size* of distributive and employment swings around that baseline.

3.1 Percentages, not abstractions: what the fixed point represents

The employment rate $e \equiv L/N$ is a percentage: the share of the labour force that is employed. The wage share $\omega \equiv W/Y$ is also a percentage: the share of output paid out as wages and salaries. A fixed point (e^*, ω^*) is therefore not a metaphysical equilibrium but a baseline pair of percentages at which, *given the prevailing regime*, neither share has a systematic tendency to rise or fall. This baseline matters because it separates two analytically distinct questions.

First, *position*: where in the (e, ω) plane the economy tends to revolve, i.e. the baseline employment percentage and the baseline wage-share percentage.

Second, *amplitude*: how far the cycle swings away from that baseline in percentage points. In the canonical Goodwin closure, amplitude is not selected by the model; it is inherited from initial conditions or shocks. That is not a technical nuisance. It is an interpretive fact: without additional institutional structure, the model can generate endogenous motion but cannot explain why some historical episodes display small distributive oscillations while others exhibit large swings that reconfigure bargaining, expectations, and organisation.

3.2 Solving for the interior fixed point

Consider the baseline dynamics given by (1) and (3). An interior fixed point (e^*, ω^*) satisfies $e^* > 0$ and $\omega^* > 0$, with the bracketed terms in each equation equal to zero. Setting $\dot{\omega} = 0$ in (1) yields

$$\gamma + \rho e^* - a = 0 \quad \Rightarrow \quad e^* = \frac{a - \gamma}{\rho}. \tag{6}$$

Setting $\dot{e} = 0$ in (3) yields

$$(1 - c_\pi)\sigma(1 - \omega^*) - \delta - a - n = 0 \quad \Rightarrow \quad \omega^* = 1 - \frac{\delta + a + n}{(1 - c_\pi)\sigma}. \tag{7}$$

Equations (6)–(7) show that the fixed point is jointly pinned by (i) wage-setting parameters (γ, ρ) and the productivity-growth benchmark a, and (ii) an accumulation regime summarized by the effective profit-to-investment mapping $(1-c_\pi)\sigma$ and the drag terms (δ, a, n). In other words, the baseline percentages are not purely "labour-market outcomes" or purely "macroeconomic outcomes": they are the numerical footprint of a regime that combines bargaining, investment allocation, technique, depreciation, and social reproduction.

3.3 Existence as relations of magnitude

Because e and ω are shares, a meaningful interior fixed point requires $0 < e^* < 1$ and $0 < \omega^* < 1$. These restrictions translate into transparent relations of magnitude.

First, $e^* > 0$ requires

$$a > \gamma, \tag{8}$$

while $e^* < 1$ requires

$$a - \gamma < \rho. \tag{9}$$

Together, (8)–(9) delimit the wage-setting regime in a way that is immediately readable in labour-studies language. Condition (8) states that baseline wage pressure γ cannot exceed the productivity-growth benchmark a if the model is to admit a nontrivial steady employment share; condition (9) states that the tightness sensitivity ρ must be large enough that the implied "normal" employment percentage does not exceed full employment. If γ is higher (a stronger baseline bargaining climate) or if ρ is higher (tightness translates more effectively into wage outcomes), the required employment share for distributive stabilization is lower: in (6), e^* falls when γ rises or when ρ rises.

Second, $\omega^* > 0$ requires

$$(1 - c_\pi)\sigma > \delta + a + n, \tag{10}$$

and $\omega^* < 1$ holds automatically if (10) holds with nonnegative parameters. Condition (10) is the baseline feasibility constraint for a positive wage share at a nondegenerate employment rate. It states that the regime's effective accumulation capacity out of profits, $(1 - c_\pi)\sigma$, must dominate the combined drag from depreciation, productivity growth, and labour-force growth, $\delta + a + n$. Read politically, this is the point at which "distribution" ceases to be a mere bargaining outcome and becomes a constraint: the wage share can only settle at a positive baseline if the investment allocation rule (the complement of capitalist consumption, $1-c_\pi$) and the technique regime (capital productivity, σ) are strong enough to prevent employment from being eroded by the joint pressures of replacement needs, labour-saving productivity growth, and social reproduction.

Finally, under the capital productivity convention $\sigma = \alpha/\beta$ in (4), with $\alpha \equiv Y/L$ and mechanization $\beta \equiv K/L$, the growth-rate identity (5) implies $\dot{\sigma}/\sigma = a - b$. When σ is treated as constant in the baseline, the technique regime is fixed by imposing $a = b$. This keeps the analysis focused: the present section treats technique as a regime parameter so that power can be read first in the distribution–employment feedback, before we endogenize technique or institutional change later in the paper.

3.4 Local geometry and the baseline cycle

To summarize local dynamics, write the system as

$$\dot{e} = e\Big((1 - c_\pi)\sigma(1 - \omega) - \delta - a - n\Big), \tag{11}$$

$$\dot{\omega} = \omega\big(\gamma + \rho e - a\big). \tag{12}$$

The Jacobian evaluated at (e^*, ω^*) is

$$J^* = \begin{pmatrix} 0 & -(1 - c_\pi)\sigma\, e^* \\ \rho\,\omega^* & 0 \end{pmatrix}. \tag{13}$$

Under the interior conditions above, $\det(J^*) = (1 - c_\pi)\sigma\,\rho\, e^*\omega^* > 0$ and $\mathrm{tr}(J^*) = 0$, implying purely imaginary eigenvalues. In the canonical closure, the fixed point is therefore locally center-like: the model generates closed motion around (e^*, ω^*) rather than local damping or local explosion. This property is analytically useful because it isolates the feedback logic, but it also reveals what is missing for empirical realism: the baseline cycle has motion, yet no internal criterion for why fluctuations should be small, large, persistent, damped, or regime-dependent.

3.5 Amplitude, percentage-point swings, and power as a relation of magnitude

If e^* and ω^* describe the *baseline* percentages, amplitude describes the *political size* of cyclical conflict. Since both variables are shares, amplitude is most transparently discussed in percentage points. For a window T (e.g. after a burn-in), define

$$\mathcal{A}_e \equiv \max_{t\in T} e(t) - \min_{t\in T} e(t), \tag{14}$$

$$\mathcal{A}_\omega \equiv \max_{t\in T} \omega(t) - \min_{t\in T} \omega(t). \tag{15}$$

In a labour-studies register, $\mathcal{A}_e$ is the swing in employment-rate percentage points across the cycle (how large the reserve army becomes and then contracts), while $\mathcal{A}_\omega$ is the swing in wage-share percentage points (how large labour's claim on output expands and then retreats). These swings are not innocent: large amplitude cycles reorganize the field of bargaining by changing risk, expectations, and the credibility of collective action. A five-point movement in the wage share is not just a data point; it reshapes what employers and workers consider feasible, and it interacts with organisational endurance in a way that small oscillations often do not.

The canonical Goodwin system, however, does not select a unique amplitude. Different initial conditions produce different values of $\mathcal{A}_e$ and $\mathcal{A}_\omega$ while preserving the same qualitative rotation around (e^*, ω^*). This matters for how "power" is read in the model. The baseline already connects tightness to distribution via ρ and profitability to employment via σ and c_π, but it cannot explain why distributive conflict should remain within narrow bounds or erupt into large swings. That gap is precisely where an explicit representation of working-class power becomes analytically necessary.

Near the fixed point, the linearized system clarifies how parameters shape the *geometry* of amplitude, even if they do not select it. Define deviations

$$x(t) \equiv e(t) - e^*, \qquad y(t) \equiv \omega(t) - \omega^*. \tag{16}$$

The linearization implied by (11)–(12) yields

$$\dot{x} = -\big((1 - c_\pi)\sigma e^*\big)\, y, \tag{17}$$

$$\dot{y} = \big(\rho\,\omega^*\big)\, x, \tag{18}$$

which implies oscillatory motion with local frequency

$$\Omega \equiv \sqrt{(1 - c_\pi)\sigma\,\rho\,e^*\omega^*}. \tag{19}$$

Most importantly for the present argument, the *relative* size of fluctuations in distribution and employment near the fixed point is governed by a simple magnitude relation:

$$\frac{\mathrm{Amp}(y)}{\mathrm{Amp}(x)} = \sqrt{\frac{\rho\,\omega^*}{(1 - c_\pi)\sigma\,e^*}}. \tag{20}$$

Equation (20) is a compact translation of a labour-studies intuition into model terms. When the tightness-to-wage transmission channel is stronger (larger ρ), distributive conflict tends to express itself more in wage-share swings for a given employment swing. When the profit-to-accumulation mapping is stronger (larger $(1 - c_\pi)\sigma$), cyclical adjustment tends to express itself more in employment swings for a given wage-share swing. The cycle's geometry thus encodes relations of power as relations of magnitude across two percentage variables: the question is not only *whether* labour gains occur, but *how large* they are relative to the employment movements that discipline or enable them.

Figures 1 and 2 illustrate these points visually. In the phase portrait, trajectories with different initial conditions trace distinct closed orbits around the same fixed point, corresponding to different amplitudes. In the time paths, the same difference appears as larger or smaller swings in wage share and employment rate around their baseline values.

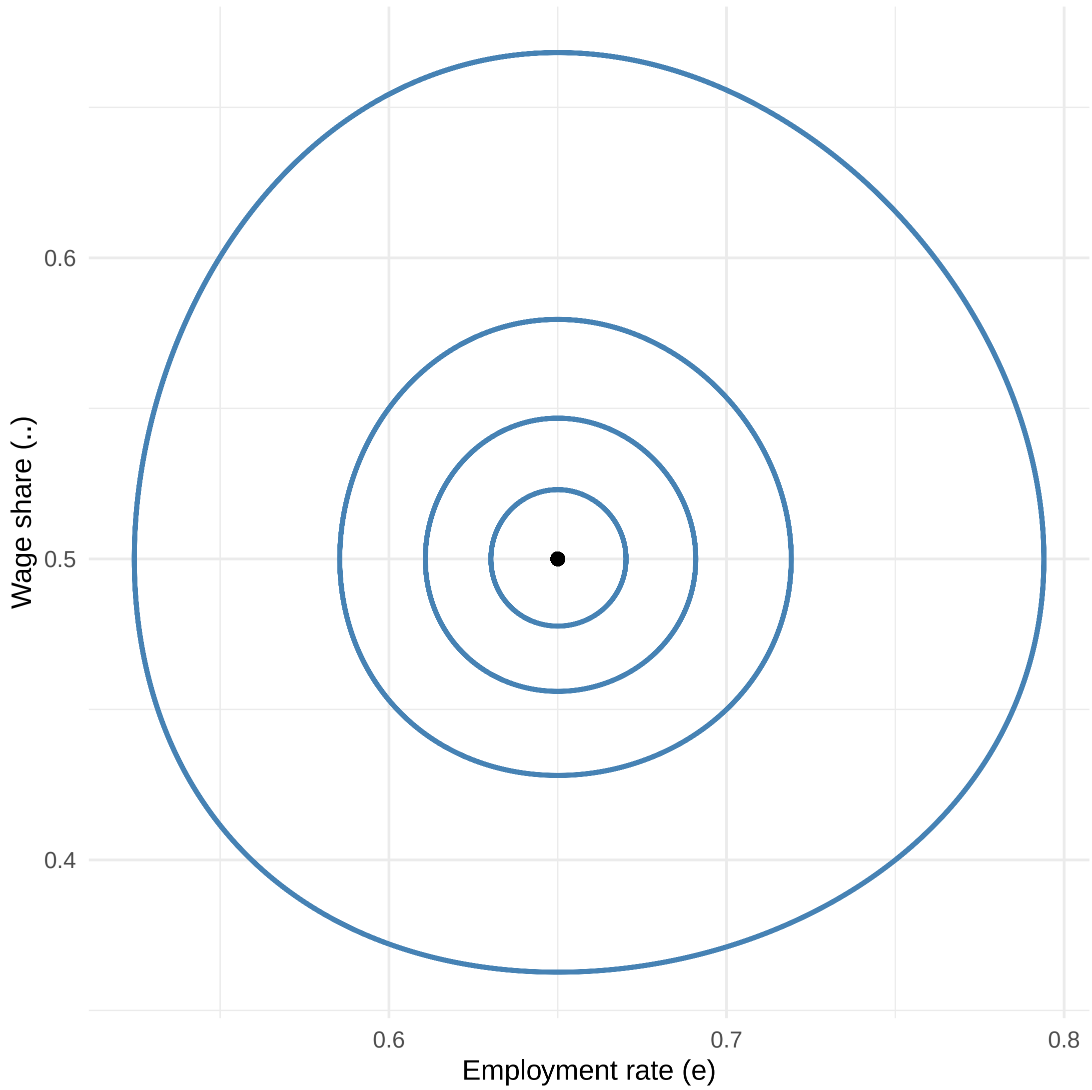

Figure 1: Phase portrait of the canonical Goodwin dynamics in (e, ω) space for multiple initial conditions. Each trajectory is a closed orbit around the interior fixed point, illustrating that amplitude is not selected by a unique limit cycle in the baseline closure.

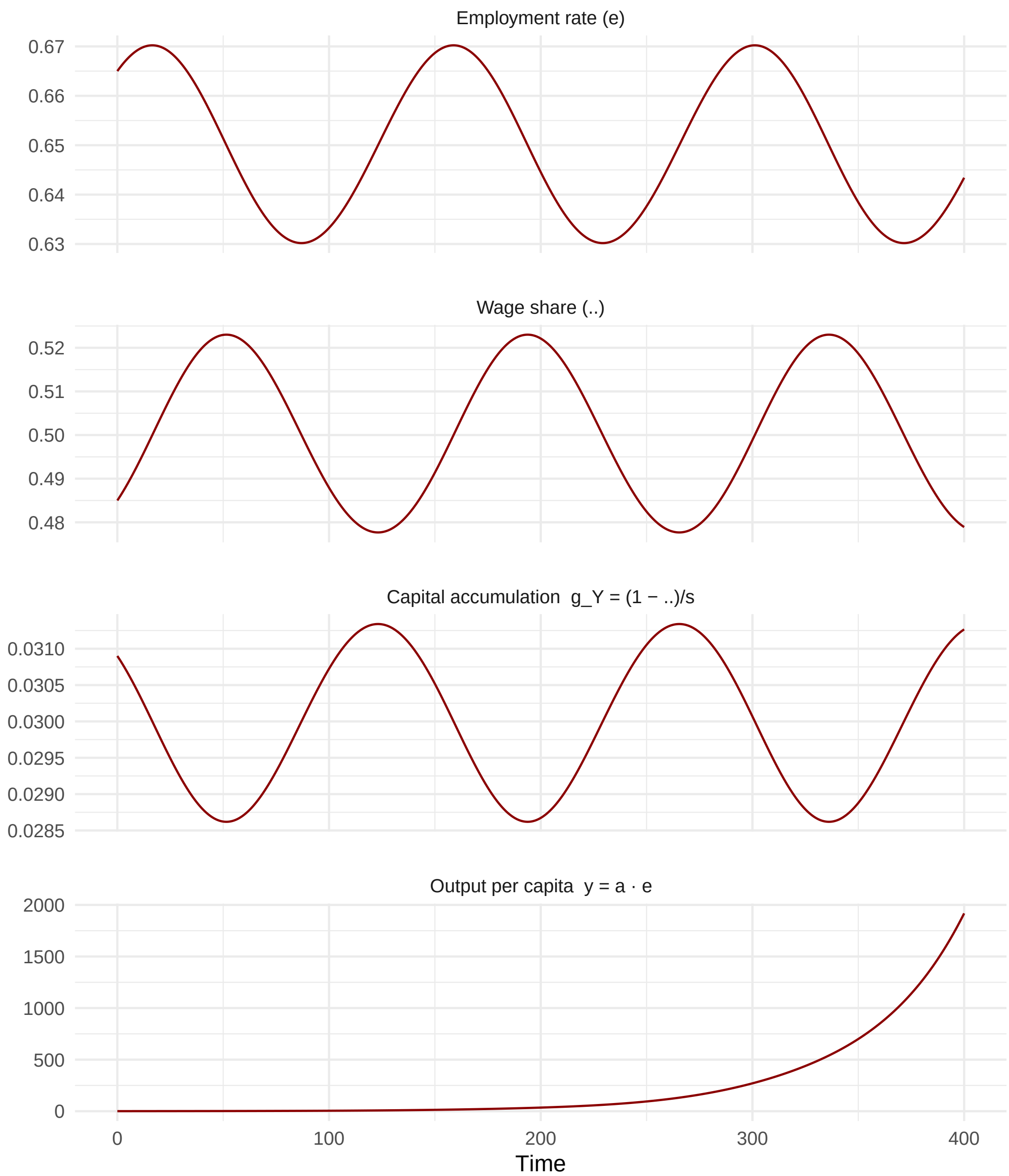

Figure 2: Time paths of employment rate $e(t)$ and wage share $\omega(t)$ in the canonical Goodwin model. Dotted lines indicate the fixed point values (e^*, ω^*) from (6)–(7). Different initial conditions generate different percentage-point amplitudes.

3.6 Working-class power in baseline magnitudes, and what the baseline cannot say

Even before introducing an explicit measure of associational power, the baseline fixed point identifies two distinct power statements expressed as relations of magnitude.

First, wage-setting power operates through the mapping from tightness to distribution in (1). The implied steady employment percentage in (6) depends on the gap $a - \gamma$ and the sensitivity ρ. A higher γ (stronger baseline wage pressure) or a higher ρ (stronger tightness-to-wage transmission) lowers e^*, meaning that the wage share can be stabilized or increased at lower employment rates. In this sense, wage-setting institutions and coordination affect the *tightness threshold* required for distributive gains.

Second, accumulation feasibility operates through (7) and its existence constraint (10).

Holding the wage-setting regime fixed, a higher capitalist consumption propensity c_π lowers the feasible wage share by reducing the fraction of surplus mobilized for accumulation demand, while a higher capital productivity σ raises it by strengthening the growth impulse per unit of profit share. In this sense, distributive outcomes are constrained not only by bargaining but by the regime governing the allocation of surplus between accumulation and other uses.

Third, and this is the point that motivates the next section, the baseline model cannot explain why the amplitude of distributive conflict should be small or large, why cycles should be damped, or why their geometry should shift across institutional regimes. In the baseline, amplitude is inherited rather than produced; yet amplitude is exactly where working-class power often becomes visible as endurance, capacity for coordination, and the ability to turn tightness into sustained distributive gains rather than episodic spikes. The next section therefore introduces a disciplined way to represent associational power—with union affiliation as a proxy—as a shifter of the wage-pressure mapping in (1), while preserving the baseline accumulation block in (3) to avoid confounding channels.

4 Union affiliation in the wage-setting block: intercept power, slope power, and canonical consequences

This section uses the canonical Goodwin system to clarify what it means to "introduce union affiliation" in a minimal, one-equation way. The objective is not to claim that unions only matter for wage-setting, but to begin with a disciplined mapping: union affiliation is treated as a proxy for associational power, and associational power is allowed to condition the Phillips/bargaining relation while the accumulation block is held fixed. This preserves the baseline Goodwin feedback while making explicit the two distinct ways organization can alter distributive dynamics: by shifting the *baseline level* of wage pressure (intercept power) and/or by changing the *tightness-to-wage transmission* (slope power).

4.1 Setup: a union-conditioned Phillips mapping

Let $\nu \in [0, 1]$ denote union affiliation (or union density/affiliation rate, depending on the dataset), reserving $s_\pi = 1 - c_\pi$ for profit saving. The baseline wage-share dynamic in (1) is generalized to allow the intercept and slope of wage pressure to depend on ν:

$$\dot{\omega} \;=\; \omega\Big(\gamma(\nu) + \rho(\nu)e - a\Big). \tag{21}$$

The employment-rate equation remains the baseline accumulation closure:

$$\dot{e} \;=\; e\Big((1 - c_\pi)\sigma(1 - \omega) - \delta - a - n\Big). \tag{22}$$

As long as union affiliation enters only through (21), the interior fixed point inherits a simple structure: unions shift the baseline employment percentage but do not mechanically change the baseline wage-share percentage implied by the accumulation regime. Formally,

$$e^*(\nu) \;=\; \frac{a - \gamma(\nu)}{\rho(\nu)}, \tag{23}$$

while

$$\omega^* \;=\; 1 - \frac{\delta + a + n}{(1 - c_\pi)\sigma}. \tag{24}$$

Equation (23) is the key interpretive object: it states that union affiliation matters first by changing the *tightness threshold* at which distributive conflict stabilizes or reverses.

4.2 Intercept power: unions raise baseline wage pressure

A first interpretation of associational power is "floor" power: organization raises baseline wage pressure even when employment is not exceptionally high. This corresponds to a union-conditioned intercept, with $\gamma'(\nu) > 0$ and $\rho(\nu) = \rho$ constant. A simple linear form is

$$\gamma(\nu) \;=\; \gamma_0 + \gamma_1 \nu, \qquad \gamma_1 > 0. \tag{25}$$

Substituting (25) into (23) yields

$$e^*(\nu) \;=\; \frac{a - \gamma_0 - \gamma_1 \nu}{\rho}. \tag{26}$$

Equation (26) implies that higher union affiliation lowers the baseline employment rate required to prevent the wage share from drifting downward: wage-share defense and expansion become possible at lower levels of tightness. In labour-studies terms, intercept power makes distributive gains less dependent on booms.

A second consequence concerns the *tempo* of endogenous oscillations. In the canonical closure, the local cycle frequency around the interior fixed point depends on e^* and ω^*. With ω^* fixed by (24), a lower $e^*(\nu)$ implies a lower local frequency, meaning that intercept power slows the pace at which employment and distribution chase each other around the baseline point (holding other parameters fixed). This will matter when we later discuss how institutional settings can reshape both the location and persistence of distributive cycles.

4.3 Slope power: unions strengthen the transmission from tightness to distribution

A second interpretation of associational power is transmission power: organization increases the extent to which tight labour markets translate into wage-share gains by coordinating wage claims and limiting undercutting. This corresponds to a union-conditioned slope, with $\rho'(\nu) > 0$ and $\gamma(\nu) = \gamma$ constant. A simple linear form is

$$\rho(\nu) \;=\; \rho_0 + \rho_1 \nu, \qquad \rho_1 > 0. \tag{27}$$

Substituting (27) into (23) yields

$$e^*(\nu) \;=\; \frac{a - \gamma}{\rho_0 + \rho_1 \nu}. \tag{28}$$

Equation (28) also lowers the baseline employment threshold when union affiliation rises, but the mechanism is different: unions do not raise wage pressure "everywhere"; they increase the tightness-to-wage transmission.

The canonical Goodwin model also delivers a sharper distinction between slope and intercept effects. Using the fixed-point condition $\gamma + \rho(\nu)e^*(\nu) = a$ implied by (21), the product $\rho(\nu)e^*(\nu)$ is pinned at $a - \gamma$. As a result, slope power does not change the local cycle frequency in the baseline closure (holding γ fixed); instead it reshapes the *geometry* of the cycle by changing how much of cyclical adjustment expresses itself in wage-share swings relative to employment swings. In short, slope power alters the distributive *expression* of the cycle more than its tempo.

4.4 Intercept and slope jointly: threshold, tempo, and expression

In many historical settings, union affiliation plausibly affects both baseline wage pressure and the transmission of tightness into distributive outcomes. With $\gamma'(\nu) > 0$ and $\rho'(\nu) > 0$, (23) implies

$$e^*(\nu) \;=\; \frac{a - \gamma(\nu)}{\rho(\nu)}. \tag{29}$$

This joint specification clarifies a useful decomposition for the empirical and interpretive agenda that follows. Intercept power shifts the cycle's *tempo* through the term $a - \gamma(\nu)$, while slope power primarily shapes the cycle's *expression* by strengthening the mapping from tightness into wage-share change. Both channels lower the tightness threshold for distributive stabilization, but they correspond to distinct modes of exercising associational power.

4.5 Canonical caveat: the baseline does not select amplitude

The preceding comparative statics are intentionally "canonical": they preserve the closed-orbit structure of the baseline Goodwin model. Consequently, the model still does not select a unique amplitude for the cycle. Different initial conditions can generate different percentage-point swings in employment and wage share around the same fixed point. This caveat is not merely technical. In labour-studies terms, amplitude is often where power becomes visible as endurance, coordination capacity, and institutional persistence: small oscillations and large swings can entail qualitatively different political dynamics. For this reason, introducing union affiliation through (21) is best understood as a first step that clarifies *where* and *how* organization enters the distributive feedback; subsequent sections can then ask what additional institutional mechanisms are required to account for damping, regime dependence, or amplitude selection in historically observed cycles.

5 Associational power beyond affiliation: Phillips-curve channels and the Hopf institutional lever

This section consolidates the paper's minimal institutionalization strategy. We treat working-class associational power as a *configuration* rather than a single statistic: union affiliation matters, but so do fragmentation across unions, the scale of bargaining units, and the degree of wage dispersion that shapes solidarity. The purpose is twofold. First, to show how these organisational features can be represented in a Goodwin framework without confounding channels, by conditioning only the wage-setting (Phillips/bargaining) relation while holding the accumulation block fixed. Second, to show why this is the natural entry point for a Hopf-bifurcation analysis: institutions can shift not only the location and geometry of distributive cycles, but also their stability and persistence.

5.1 From organisational features to an effective coordination index

Let $\nu \in [0, 1]$ denote union affiliation (or union density/affiliation rate), reserving $s_\pi = 1 - c_\pi$ for profit saving. Let U denote the number of unions (a fragmentation proxy), let $\bar{s}$ denote average union size (a bargaining-unit scale proxy), and let Δ denote a skill wage differential (a solidarity proxy). These objects can be treated as slow-moving institutional features or observed covariates; their role here is conceptual: they shift the mapping from labour-market tightness to distributive pressure.

To keep the model disciplined, we summarize these features in an effective coordination index χ:

$$\chi \;\equiv\; \chi(\nu, U, \bar{s}, \Delta), \qquad \chi \in [0, 1]. \tag{30}$$

One transparent specification is

$$\chi(\nu, U, \bar{s}, \Delta) \;=\; \Lambda\big(\eta_0 + \eta_\nu \ln\big(1+\nu\big) - \eta_U \ln\big(1+U\big) + \eta_s \ln\big(1+\bar{s}\big) - \eta_\Delta \ln\big(1+\Delta\big)\big)\,, \tag{31}$$

where $\Lambda(x) \equiv 1/(1 + e^{-x})$ and $\eta_\nu, \eta_U, \eta_s, \eta_\Delta > 0$. Equation (31) encodes three labour-studies claims in reduced form: higher affiliation and larger average union size raise effective coordination; more unions (higher fragmentation) lowers it; and larger skill wage differentials erode solidarity and thus reduce effective coordination. The section does not require this exact parameterization; what matters is that χ operationalizes associational power as a coherent organisational configuration rather than a single scalar.

5.2 One-equation discipline: conditioning only the wage-setting block

The wage-share dynamic generalizes the baseline Phillips/bargaining mapping by allowing its intercept and slope to depend on χ:

$$\dot{\omega} \;=\; \omega\Big(\gamma(\chi) + \rho(\chi)e - a\Big). \tag{32}$$

The employment-rate equation remains the baseline accumulation closure:

$$\dot{e} \;=\; e\Big((1-c_\pi)\sigma(1-\omega) - \delta - a - n\Big). \tag{33}$$

As long as associational power enters only through (32), the interior fixed point retains a simple structure. The baseline employment percentage required for distributive stabilization is

$$e^*(\chi) \;=\; \frac{a-\gamma(\chi)}{\rho(\chi)}, \tag{34}$$

while the baseline wage-share percentage remains pinned by accumulation feasibility:

$$\omega^* \;=\; 1 - \frac{\delta + a + n}{(1-c_\pi)\sigma}. \tag{35}$$

Equation (34) is the central object for the labour-studies interpretation. Higher effective coordination lowers the tightness threshold required for wage-share gains: organization and solidarity substitute for tightness as a condition for distributive effectiveness. By contrast, the baseline wage share in (35) is governed by the accumulation regime, which is held fixed here to avoid confounding channels.

5.3 Three canonical Phillips-curve channels of associational power

Effective coordination can alter the Phillips/bargaining relation through two conceptually distinct channels, corresponding to the two modes of associational power identified earlier: "floor" power and "transmission" power. The three canonical cases below differ only in how χ enters $\gamma(\cdot)$ and $\rho(\cdot)$.

Case 1: Intercept (floor) power. Organization raises baseline wage pressure even when employment is not exceptionally high:

$$\gamma(\chi) = \gamma_0 + \gamma_1\chi, \qquad \rho(\chi) = \rho_0, \qquad \gamma_1 > 0. \tag{36}$$

In this case,

$$e^*(\chi) = \frac{a - \gamma_0 - \gamma_1\chi}{\rho_0}, \tag{37}$$

so higher χ lowers the baseline employment threshold for distributive stabilization. In labour-studies terms, associational power substitutes for tightness: wage-share defence becomes feasible at lower employment rates.

Case 2: Slope (transmission) power. Organization strengthens the translation of tightness into wage-share pressure by coordinating claims and limiting undercutting:

$$\gamma(\chi) = \gamma_0, \qquad \rho(\chi) = \rho_0 + \rho_1\chi, \qquad \rho_1 > 0. \tag{38}$$

In this case,

$$e^*(\chi) = \frac{a - \gamma_0}{\rho_0 + \rho_1\chi}. \tag{39}$$

Again, higher χ lowers the threshold, but now through a distinct mechanism: it increases the tightness-to-wage transmission. In the canonical Goodwin closure, this channel is naturally interpreted as reshaping the *expression* of conflict, shifting cyclical adjustment toward larger wage-share swings relative to employment swings for a given disturbance.

Case 3: Joint channel. Effective coordination affects both floor power and transmission power:

$$\gamma(\chi) = \gamma_0 + \gamma_1\chi, \qquad \rho(\chi) = \rho_0 + \rho_1\chi, \qquad \gamma_1 > 0,\ \rho_1 > 0. \tag{40}$$

In this case,

$$e^*(\chi) = \frac{a - \gamma_0 - \gamma_1\chi}{\rho_0 + \rho_1\chi}. \tag{41}$$

The joint channel clarifies a decomposition that will be useful empirically. Intercept power lowers the tightness threshold by raising baseline wage pressure; slope power lowers the threshold and reshapes how tightness translates into distributive outcomes. Together, they summarize how organisational configuration moves the baseline employment percentage at which distributive dynamics change sign.

5.4 From level shifts to regime shifts: the Hopf institutional lever

The Phillips-curve channels above shift the location and geometry of distributive dynamics, but the canonical Goodwin closure remains locally center-like and does not, by itself, select amplitude or generate damped versus persistent cycles. Yet amplitude and persistence are precisely where working-class power often becomes visible as endurance, coordination capacity, and institutional durability. To represent institutions as regime makers rather than mere shifters of baseline percentages, we introduce a stability lever that depends on effective coordination.

A minimal representation is to add a local stabilization/destabilization term to the wage-share equation:

$$\dot{\omega} \;=\; \omega\Big(\gamma(\chi) + \rho(\chi)e - a\Big) \;-\; \mu(\chi)\,(\omega - \omega^*), \tag{42}$$

where $\mu(\chi)$ captures whether the institutional configuration dampens ($\mu(\chi) > 0$) or amplifies ($\mu(\chi) < 0$) deviations of the wage share from its baseline value.[1]

A parsimonious "institutional flip" is

$$\mu(\chi) = \zeta(\chi - \chi_0), \qquad \zeta \neq 0, \tag{43}$$

so that χ_0 defines a regime boundary. Under mild regularity conditions, a Hopf bifurcation occurs when the local stability of the interior fixed point changes sign while oscillatory structure is preserved. In this representation, the boundary is captured by

$$\mu(\chi_0) = 0 \quad\Longleftrightarrow\quad \chi = \chi_0. \tag{44}$$

[1] Equation (42) is intentionally local: it represents the idea that wage-setting can become more rule-bound and self-correcting under some institutional configurations and more escalatory and self-reinforcing under others. Alternative microfoundations are possible. The point is to separate (i) location effects governed by $\gamma(\chi), \rho(\chi)$ from (ii) stability effects governed by $\mu(\chi)$.

Equation (44) is the central institutional claim. Changes in affiliation, fragmentation, bargaining-unit scale, or wage solidarity can move the system across a stability boundary: distributive conflict becomes either locally self-correcting (damped) or locally self-reproducing (persistent oscillations). Because χ aggregates multiple organisational features, the boundary is not a single threshold in affiliation alone. Two societies with the same affiliation rate ν can lie on different sides of (44) if one has high fragmentation U and a large skill premium Δ, while the other has larger bargaining units $\bar{s}$ and stronger wage compression.

5.5 Closing interpretation: structural power, associational power, and amplitude

The consolidated framework yields a clean division of labour between structural and associational power. Structural power operates through tightness e: it is the objective labour-market terrain on which conflict is fought. Associational power operates through χ: it conditions how tightness translates into distribution, and whether distributive conflict is damped or persistent. The Phillips-curve channels (36)–(40) determine the *tightness threshold* for distributive gains and the *expression* of conflict in wage-share versus employment swings. The Hopf lever (42)–(44) determines whether those dynamics are self-correcting or self-reproducing, providing a disciplined route to analyzing amplitude and persistence as institutional outcomes rather than as artifacts of initial conditions.

6 Operationalizing empirical hypotheses

The theoretical framework is designed to be empirically operational in a direct sense: it maps measurable features of working-class organization into distinct objects in the Goodwin wage-setting block (intercept, slope, and stability), and it yields observable implications for (i) the baseline location of employment and distribution, (ii) the geometry of co-movement between e and ω, and (iii) regime changes in the persistence of distributive cycles. This section states the empirical counterparts of the model objects and derives a set of testable hypotheses using Chilean time-series data on employment, wage share, and union structure.

6.1 Measurement and empirical counterparts

We observe the employment rate e_t and wage share ω_t as bounded percentage variables. To align with the model's multiplicative structure, we work with log changes for dynamics and levels for state dependence. Define

$$\Delta \ln \omega_t \equiv \ln \omega_t - \ln \omega_{t-1}, \qquad \Delta \ln e_t \equiv \ln e_t - \ln e_{t-1}. \tag{45}$$

Labour productivity growth is measured as

$$a_t \equiv \Delta \ln \alpha_t, \qquad \alpha_t \equiv \frac{Y_t}{L_t}. \tag{46}$$

Capital productivity is measured as

$$\sigma_t \equiv \frac{Y_t}{K_t}, \tag{47}$$

or equivalently we use K_t/Y_t depending on data availability and measurement quality.

Associational power is not proxied by union affiliation alone. We therefore construct a coordination/solidarity index χ_t from: union affiliation (density) ν_t, number of unions U_t, average union size $\bar{s}_t$, and a skill wage differential Δ_t (skill premium). We define

$$\chi_t \equiv \chi(\nu_t, U_t, \bar{s}_t, \Delta_t) \in [0, 1], \tag{48}$$

with a baseline implementation

$$\chi_t = \Lambda(\eta_0 + \eta_\nu \ln(1+\nu_t) - \eta_U \ln(1+U_t) + \eta_s \ln(1+\bar{s}_t) - \eta_\Delta \ln(1+\Delta_t))\,, \tag{49}$$

where $\Lambda(x) = 1/(1+e^{-x})$ and $\eta_\nu, \eta_U, \eta_s, \eta_\Delta > 0$. The empirical role of χ_t is to capture how organizational configuration conditions wage-setting and stability. Results are checked for robustness to alternative constructions (e.g. principal components, z-score indices, or excluding one input at a time).

6.2 Empirical wage-setting equation and the Phillips mapping

The model's wage-share equation implies an empirically interpretable reduced form. From $\dot{\omega}/\omega = \gamma(\chi) + \rho(\chi)e - a$, the discrete-time analogue is

$$\Delta \ln \omega_t = \gamma(\chi_{t-1}) + \rho(\chi_{t-1})\, e_{t-1} - a_{t-1} + \mathbf{x}'_{t-1}\boldsymbol{\theta} + \varepsilon^\omega_t, \tag{50}$$

where $\mathbf{x}_{t-1}$ includes controls and scaling terms motivated by the model (e.g. output growth, inflation regime indicators, external constraint proxies, policy shifts) and where lagging is used to reduce mechanical simultaneity.

To test the canonical channels of associational power, we estimate three nested cases.

Intercept power (floor power).

$$\gamma(\chi_{t-1}) = \gamma_0 + \gamma_1 \chi_{t-1}, \qquad \rho(\chi_{t-1}) = \rho_0, \tag{51}$$

which yields

$$\Delta \ln \omega_t = \gamma_0 + \gamma_1 \chi_{t-1} + \rho_0 e_{t-1} - a_{t-1} + \mathbf{x}'_{t-1}\boldsymbol{\theta} + \varepsilon^\omega_t. \tag{52}$$

Slope power (transmission power).

$$\gamma(\chi_{t-1}) = \gamma_0, \qquad \rho(\chi_{t-1}) = \rho_0 + \rho_1 \chi_{t-1}, \tag{53}$$

which yields the interaction form

$$\Delta \ln \omega_t = \gamma_0 + \rho_0 e_{t-1} + \rho_1(\chi_{t-1} e_{t-1}) - a_{t-1} + \mathbf{x}'_{t-1}\boldsymbol{\theta} + \varepsilon^\omega_t. \tag{54}$$

Joint channel.

$$\gamma(\chi_{t-1}) = \gamma_0 + \gamma_1 \chi_{t-1}, \qquad \rho(\chi_{t-1}) = \rho_0 + \rho_1 \chi_{t-1}, \tag{55}$$

which yields

$$\Delta \ln \omega_t = \gamma_0 + \gamma_1 \chi_{t-1} + \rho_0 e_{t-1} + \rho_1(\chi_{t-1} e_{t-1}) - a_{t-1} + \mathbf{x}'_{t-1}\boldsymbol{\theta} + \varepsilon^\omega_t. \tag{56}$$

These specifications allow direct operational tests: intercept power corresponds to $\gamma_1 > 0$; slope power corresponds to $\rho_1 > 0$. The framework also implies a derived threshold object, $e^{t=(a_t - \gamma(\chi_t))/\rho(\chi_t)}$, which can be tracked over time as an estimated "tightness requirement" for wage-share stabilization.

6.3 Growth and accumulation block: controls, scaling, and disciplined minimality

The methodological strategy is deliberately minimal: associational power is introduced first in wage-setting, while the accumulation block is treated as a control-and-scaling environment rather than a simultaneous institutional channel. In reduced form, the model suggests that employment growth depends on profit-led accumulation scaled by capital productivity:

$$\Delta \ln e_t = \lambda_0 + \lambda_1(1-\omega_{t-1})\sigma_{t-1} + \lambda_2 \mathbf{z}'_{t-1} + \varepsilon^e_t, \tag{57}$$

where $\mathbf{z}_{t-1}$ includes proxies for depreciation replacement needs, labour-force growth, and macro trend components (or their empirical counterparts). The purpose of (57) is not to claim identification of the full accumulation mechanism, but to ensure that the Phillips estimates are not confounding wage-setting with omitted growth/technique shifts.

6.4 From level effects to regime effects: a testable Hopf boundary

The Hopf interpretation turns an institutional narrative into an empirically testable claim: the local dynamics of the (e, ω) system change stability as the organizational configuration changes. Operationally, this can be tested in two complementary ways.

Approach 1: state-dependent local linearization (Jacobian mapping). Define the empirical state vector as $\mathbf{y}_t = (\ln e_t, \ln \omega_t)'$. Estimate a local linear law of motion in a neighborhood of time t:

$$\Delta \mathbf{y}_t = \mathbf{A}(\chi_{t-1})\big(\mathbf{y}_{t-1} - \bar{\mathbf{y}}(\chi_{t-1})\big) + \mathbf{u}_t, \tag{58}$$

where $\mathbf{A}(\chi)$ is a state-dependent Jacobian analogue, estimated via local regressions, rolling windows, or smooth transition methods. The Hopf claim corresponds to a stability flip as χ crosses a threshold: the real part of complex eigenvalues changes sign. In practice, we track a stability proxy such as

$$\mathcal{S}(\chi) \equiv \max \Re\big(\lambda_i(\mathbf{A}(\chi))\big), \tag{59}$$

and test whether $\mathcal{S}(\chi)$ crosses zero at some χ_0.

Approach 2: threshold or smooth-transition VAR in (e, ω) with χ as regime variable. Estimate a TVAR/STVAR where regimes are defined by χ:

$$\Delta \mathbf{y}_t = \mathbf{B}_0^{(r)} + \sum_{j=1}^{p} \mathbf{B}_j^{(r)} \Delta \mathbf{y}_{t-j} + \mathbf{u}_t, \qquad r = \begin{cases} 1, & \chi_{t-1} < \chi_0, \\ 2, & \chi_{t-1} \geq \chi_0, \end{cases} \tag{60}$$

or with a smooth transition weight. The operational Hopf hypothesis is then: the implied local dynamics around the regime-specific fixed point are stable in one regime and oscillatory/persistent in the other, with the threshold variable χ capturing the institutional lever. The "bifurcation" is not imposed; it is evaluated by comparing stability diagnostics across regimes and by testing the threshold statistically.

6.5 Empirical hypotheses and identification logic

The framework yields a tractable hypothesis set:

- **H1 (Intercept power):** $\gamma_1 > 0$ in (52) or (56). Higher effective coordination raises baseline wage pressure, lowering the tightness threshold e^t.
- **H2 (Slope power):** $\rho_1 > 0$ in (54) or (56). Higher effective coordination strengthens the translation of tightness into wage-share growth.

- **H3 (Threshold drift as institutional outcome):** the estimated e_t^* varies systematically with χ_t via (34) (empirical analogue), implying that institutional configuration shifts the employment percentage required for distributive stabilization.
- **H4 (Hopf/stability flip):** there exists χ_0 such that stability diagnostics $\mathcal{S}(\chi)$ in (59) cross zero or regime-specific dynamics in (60) switch from damped to persistent oscillation (or vice versa).
- **H5 (Solidarity erosion):** higher wage dispersion (higher Δ_t) reduces χ_t and thereby weakens both intercept and slope channels, pushing the system toward the damped/low-transmission regime.

The main methodological complication is endogeneity: union structure co-moves with the cycle. We address this by (i) lagging organizational variables, (ii) using regime-shift methods where institutional breaks are treated as slow-moving states rather than contemporaneous shocks, and (iii) where possible, exploiting policy and legal reforms as discrete instruments or event-study anchors [CITATION NEEDED]. Robustness checks include alternative measures of employment tightness, alternative wage-share constructions, and excluding periods with severe measurement changes.

6.6 What the framework adds empirically

Standard empirical Phillips curve work asks whether tightness predicts wage dynamics. The present framework asks a sharper question: *what organizational configuration makes tightness distributively effective, and when does that configuration alter the stability of the growth–distribution system?* In this sense, the Goodwin structure is not used as a literal data-generating process; it is used as a disciplined map from labour-power theory to estimable objects (intercept, slope, and stability) and from estimable objects to regime hypotheses (Hopf boundary) that can be confronted with data.

7 Data: Chilean growth, distribution, and union structure (1932–2010)

Our empirical analysis uses an annual Chilean time-series dataset covering the period 1932–2010 (79 observations). The dataset is organized as a single country panel in long-run time, with two sheets: a `data` sheet containing the series and a `legend` sheet documenting variable definitions and sources. For transparency and reproducibility, the SHA-256 checksum of the Excel file used in this draft is: `c3a043539c2dddd79a623a4b4b2f70fe1bdbfd30096ba889ef6451eee73d2aea`.

7.1 Core state variables

The two canonical Goodwin state variables are measured as bounded percentage variables (stored as shares in $[0,1]$):

- Employment rate (e_t): measured as `emp_rate` (share of the labour force employed) from Clio-Lab PUC (https://cliolab.economia.uc.cl/). The dataset also includes `unemp_rate`, the unemployment rate from the same source.
- Wage share (ω_t): measured as `w_sh` (labour share of income/output) from Astorga (2023) [CITATION NEEDED].

Because both objects are shares, the empirical series are directly interpretable in percentage points (e.g. $\omega = 0.50$ corresponds to a 50% wage share). This is useful for labour-studies interpretation, but it also imposes discipline: changes are bounded and should not be modelled as if they were unbounded levels.

7.2 Union structure and associational power proxies

To move beyond a single union-density proxy, the dataset includes three complementary measures of union structure from the Repositorio Estadísticas Sindicales (`https://repositoriosindical.netlify.app/`):

- `tot_union`: total number of unions (U_t).
- `tot_af`: total number of unionized workers (affiliates) (A_t).
- `unmean_size`: mean union size ($\bar{s}_t$), constructed as affiliates per union.

Two summary measures are included explicitly (and can be exactly reproduced from the underlying counts):

$$\bar{s}_t \equiv \frac{A_t}{U_t}, \tag{61}$$

where A_t corresponds to `tot_af` and U_t to `tot_union`; and

$$\nu_t \equiv \frac{A_t}{L_t}, \tag{62}$$

where ν_t is the union affiliation (density) rate, stored as `aff_rate`, and L_t is employment (measured as `ocup`). Equation (62) makes explicit the denominator: in this dataset, affiliation is computed over employed workers rather than over the labour force.

Employment and labour force levels used for scaling are taken from Clio-Lab PUC: `ocup` (occupied/employed persons) and `lf` (labour force).

7.3 Auxiliary scaling series

The dataset is designed to support extensions that discipline the growth/distribution mapping with productivity- and technique-relevant scaling terms. In the present file, the most direct scaling objects are:

- Labour force and employment levels: `lf` and `ocup` (Clio-Lab PUC), which permit cross-checks of rate constructions and facilitate alternative tightness measures.
- Unemployment and employment rates: `unemp_rate` and `emp_rate` (Clio-Lab PUC), which provide the bounded state variable e_t as a directly observed share.

Variables such as output growth, labour productivity growth a_t, and capital productivity $\sigma_t = Y_t/K_t$ (or its inverse K_t/Y_t) are conceptually central to the model and will be merged from national accounts and capital-stock sources in the next data-build step [CITATION NEEDED]. We keep those series outside this file at this stage to maintain a clear separation between (i) labour-market and union-structure inputs and (ii) the macro accounting layer used for controls and scaling.

7.4 Variable inventory and sources

Table 1 summarizes the variables, definitions, and sources as documented in the dataset legend.

Table 1: Chilean annual time-series dataset (1932–2010): variables and sources

Series	Meaning	Source	Link / Reference
w_sh	Wage share (ω_t)	Astorga (2023)	[CITATION NEEDED]
emp_rate	Employment rate (e_t)	Clio-Lab PUC	https://cliolab.economia.uc.cl/
unemp_rate	Unemployment rate	Clio-Lab PUC	https://cliolab.economia.uc.cl/
lf	Labour force level	Clio-Lab PUC	https://cliolab.economia.uc.cl/
ocup	Employment level	Clio-Lab PUC	https://cliolab.economia.uc.cl/
tot_union	Number of unions (U_t)	Repositorio Sindical	https://repositoriosindical.netlify.
tot_af	Unionized workers (A_t)	Repositorio Sindical	https://repositoriosindical.netlify.
unmean_size	Mean union size ($\bar{s}_t$)	Repositorio Sindical	https://repositoriosindical.netlify.
aff_rate	Union affiliation rate (ν_t)	Repositorio Sindical	https://repositoriosindical.netlify.

7.5 Notes on construction and interpretation

Three points matter for interpretation.

First, the union variables separate *scale* and *fragmentation*. A given affiliation rate ν_t can correspond to many small unions (high U_t, low $\bar{s}_t$) or fewer large unions (low U_t, high $\bar{s}_t$). This distinction is central to our modelling strategy because it allows us to distinguish associational "capacity" (resources and coordination) from simple membership prevalence.

Second, the dataset treats employment and wage share as shares in $[0, 1]$. In estimation and simulation, we therefore interpret movements in percentage points and use transformations that respect boundedness (e.g. log changes for dynamics, level interactions for state dependence), rather than treating these series as unbounded levels.

Third, the union affiliation rate `aff_rate` is constructed over employed workers `ocup` as in (62). This choice is substantively meaningful: it measures the organizational reach over those currently inside employment, rather than over the full labour force. Where relevant, we will check robustness to alternative denominators (e.g. labour force) when additional series are merged in later steps.

8 Preliminary empirical findings for Chile (1932–2010)

This section summarises a preliminary exploration of Chilean annual data on growth, distribution and union structure covering 1932–2010. The variables of interest are the employment rate (the share of the labour force that is employed, denoted e_t), the wage share (the labour share of income, ω_t), and several measures of working-class organisation. The purpose of this section is to provide descriptive evidence and motivate the more structural modelling presented later.

8.1 Data description

The sample contains 79 annual observations. The employment rate and wage share are recorded as shares in $[0, 1]$; movements are therefore interpreted in percentage points. Three organisational variables are used to proxy associational power:

- **Union affiliation rate** (ν_t): the ratio of unionised workers to employment. This measures membership prevalence.
- **Number of unions** (U_t): a measure of fragmentation.
- **Average union size** ($\bar{s}_t$): affiliates per union, interpreted as a proxy for bargaining scale. Larger average size indicates fewer, larger bargaining units.

Table 2 reports summary statistics. Means, standard deviations and extrema illustrate considerable variation, especially in organisation variables.

Table 2: Descriptive statistics for selected variables (1932–2010)

Variable	Mean	Std. dev.	Min	Max
Wage share ω_t	0.505	0.088	0.356	0.690
Employment rate e_t	0.909	0.052	0.730	0.960
Union affiliation ν_t	0.141	0.069	0.047	0.360
Average union size $\bar{s}_t$	113.16	29.64	69.22	154.22
Number of unions U_t	4574.63	2952.27	421.00	9871.00

8.2 Correlation patterns

Simple Pearson correlations highlight salient relationships. Wage share is strongly positively correlated with average union size (+0.62) and strongly negatively correlated with the number of unions (–0.74). These patterns suggest that fragmentation (many small unions) is associated with lower labour shares, whereas larger bargaining units are associated with higher labour shares. The correlation between the employment rate and wage share is weaker (+0.13), consistent with structural power but modest in magnitude. Union affiliation rate alone displays almost no correlation with the wage share (–0.02), indicating that organisational configuration matters more than membership prevalence.

8.3 Time-series visualisation

To visualise long-run dynamics, Figures 3–6 plot the wage share, employment rate, union affiliation rate and two union structure measures. All series are annual. The wage share rises through the mid-20th century, peaks in the late 1950s, declines sharply during the 1970s, and partially recovers thereafter. The employment rate fluctuates within a relatively narrow band, with slack in the early 1980s and tight labour markets in the 1990s and mid-2000s. Union density builds steadily until the early 1970s, collapses after 1973, and slowly rebuilds in the democratic era. The number of unions increases rapidly before 1973, falls sharply during the dictatorship, and grows again in the late 1980s, while average union size moves inversely, reflecting more fragmentation when there are many unions.

8.4 Preliminary regression exploration

As a first pass, ordinary least squares regressions were estimated using log changes of the wage share ($\Delta \ln \omega_t$) as the dependent variable. Regressions included lagged employment rates and various union structure variables as predictors. No model produced statistically significant coefficients. In particular, union affiliation alone did not predict wage-share growth, and interactions between employment and affiliation were not significant. These findings underscore the limitations of simple linear specifications and suggest that organisational configuration and institutional regimes need to be modelled more explicitly, as proposed by the Goodwin–Hopf framework.

8.5 Interpretation and next steps

The descriptive patterns and preliminary regressions indicate that labour share dynamics in Chile are closely linked to union structure. Fragmentation appears to depress the wage share, whereas larger bargaining units support it. Standard Phillips-curve regressions do not capture these relationships well, highlighting the need for a model that treats organisation as an institutional lever affecting both the level of distribution and the persistence of distributive cycles. The next section therefore introduces the effective coordination index and the Hopf–bifurcation approach, which together provide a disciplined framework for analysing how associational and structural power shape growth and distribution.

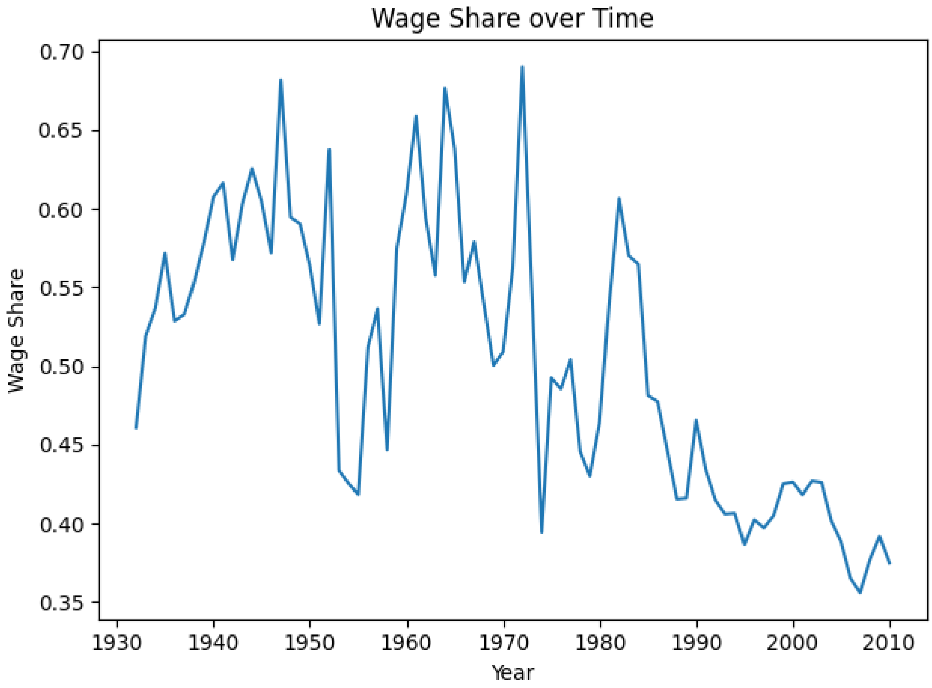

Figure 3: Wage share ω_t (share of income accruing to labour, 1932–2010). The series is plotted in levels. A peak occurs around 1958, followed by a decline during the 1970s and a partial recovery thereafter.

Figure 4: Employment rate e_t (share of labour force employed, 1932–2010). The series fluctuates between 73 % and 96 %. Notable slack occurs in the early 1980s, with recovery in the 1990s.

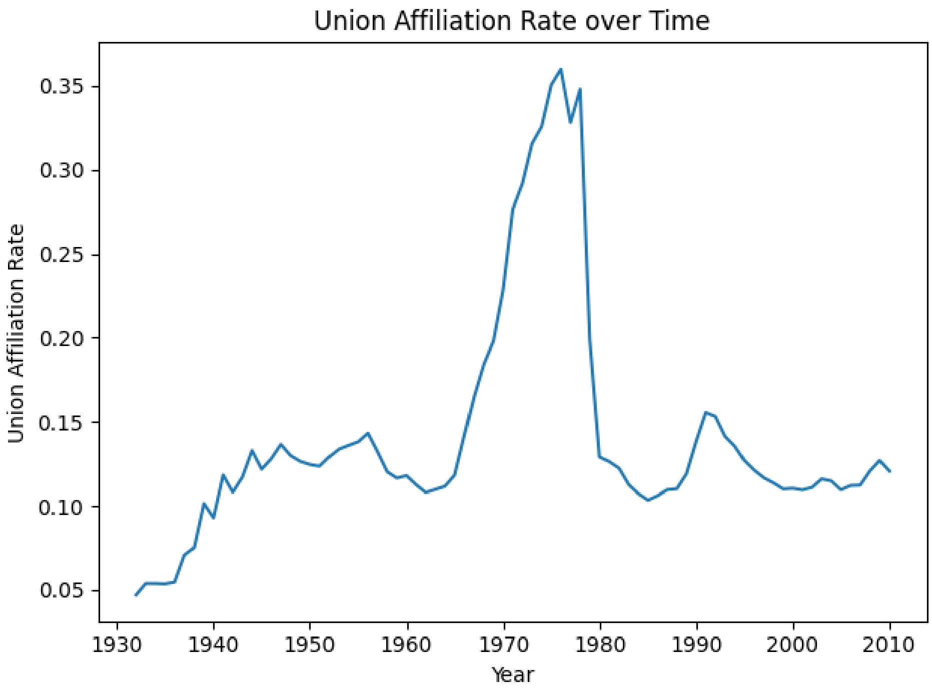

Figure 5: Union affiliation rate ν_t (share of employed workers unionised, 1932–2010). Union density rises until the early 1970s, collapses after 1973, and slowly rebuilds in the democratic era.

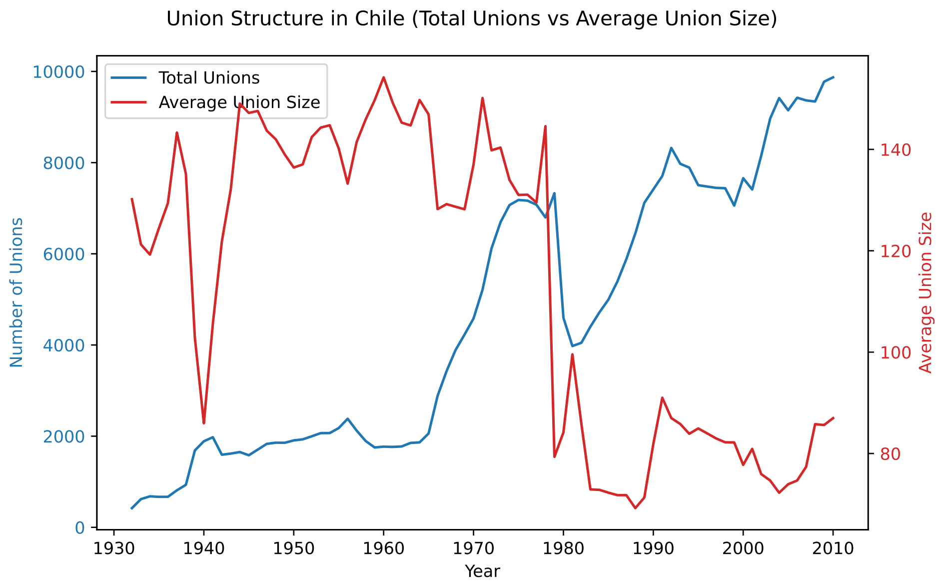

Figure 6: Union structure: number of unions U_t and average union size $\bar{s}_t$ (1932–2010). The number of unions grows rapidly prior to 1973, falls during the dictatorship, and grows again in the late 1980s. Average union size moves inversely, indicating that fragmentation and bargaining scale are dynamically linked.

9 Conclusion

This paper has argued that working-class power can and should be represented as a dynamic determinant of growth–distribution processes, rather than as a static modifier of outcomes. Using the Goodwin framework as a disciplined analytical grammar, we have shown how power operates simultaneously as an outcome of economic conditions and as a cause of their evolution. Employment and the wage share are not merely indicators; they are state variables that encode relations of magnitude through which conflict unfolds over time. Three conclusions follow.

First, the interior fixed point of the Goodwin model is best understood not as an equilibrium in the conventional sense, but as a baseline configuration of employment and distribution implied by a particular institutional–technical regime. The conditions for its existence are themselves power statements: they specify the magnitudes of productivity growth, bargaining pressure, accumulation capacity, and surplus allocation required for a positive wage share at a nondegenerate employment rate. Even before introducing explicit institutional variables, the model already embeds power in its magnitude relations.

Second, distinguishing between structural and associational power clarifies both theory and empirics. Structural power operates through labor market tightness and is already captured by employment. Associational power operates through organization and coordination, and must therefore be represented as conditioning how tightness translates into wage dynamics. By focusing initially on the wage-setting block, we isolate this channel and avoid conflating distributive mechanisms with investment or technology choices. The distinction between intercept and slope effects further refines this mapping, separating baseline wage pressure from the effectiveness of tightness in generating distributive change.

Third, introducing associational power as an institutional lever opens the door to a regime interpretation of distributive dynamics. Once stability properties depend on organizational configuration, the Hopf bifurcation theorem provides a natural language for describing institutional transitions. In this framework, damped adjustment corresponds to conflict that is locally self-correcting, while persistent oscillations correspond to conflict that reproduces itself over time. Changes in union affiliation, fragmentation, size, or wage dispersion can therefore be understood as shifts in the stability regime of capitalism, not merely as shifts in average outcomes.

The empirical exploration for Chile illustrates the promise and the limits of this approach. Descriptive patterns in wage shares, employment, and union structure are consistent with the idea that organization conditions distributive transmission. Preliminary regressions suggest that union variables matter more through interaction and coordination than through density alone. At the same time, the data underscore the difficulty of identifying dynamic regimes in finite samples, and the need for careful treatment of endogeneity, slow-moving institutions, and historical breaks.

Several extensions follow naturally. Future work can relax the fixed-technique assumption to study how conflicts over mechanization and productivity feed back into accumulation. The framework can be extended to include state policy as a separate institutional lever, or to embed the growth–distribution cycle in an open-economy constraint. Empirically, richer identification strategies and comparative cases can be used to test whether similar stability shifts occur across different institutional settings.

The broader contribution, however, is methodological. By treating working-class power as a dynamic, institutionally mediated force that shapes both the level and the persistence of distributive conflict, the paper offers a way to reconnect labor studies with macroeconomic dynamics without reducing either to the other. Growth and distribution are not just outcomes to be explained; they are the terrain on which power is exercised and contested over time.

References

Richard M. Goodwin. A growth cycle. *Socialism, Capitalism and Economic Growth*, pages 54–58, 1967.